Financial Time Series and ARCH-Class Models

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Introduction

- financial time series give information about development of prices on financial markets, e.g. about development of asset prices or prices of different currencies
- these prices are mostly recorded with high frequency, e.g. on daily basis
- the typical feature of the financial time series is the nonstationarity, but the analyses are mostly done for return series which are in general already stationary
- the main feature of return series is the time-varying variability/volatility caused probably by the situation on financial markets which are very sensitive to information of different types e.g. on political changes, changes in fiscal and monetary policy, natural catastrophes or military conflicts

Introduction

- the pioneering work in the area of modelling volatility of financial time series - autoregressive conditional heteroscedasticity (ARCH) model - was presented by Engle (1982)
- conditional variance (volatility) in ARCH model is a function of squared disturbances from previous periods and therefore enables to catch the volatility clustering, i.e. that large (small) changes tend to be followed by another large (small) changes
- Engle together with another famous econometrician Granger received in 2003 the Nobel prize in Economic Sciences „*for methods of analyzing economic time series with time-varying volatility (ARCH)*“ and „*for methods of analyzing economic time series with common trends (cointegration)*“, respectively

Introduction

- traditional approach to time series analysis – decomposition approach – decomposition into individual components (trend, seasonal, cyclical and irregular/random component)
- newer approach – Box-Jenkins ARIMA methodology
  - models AR, MA, ARMA, I, ARIMA:

    conditional mean: time-varying,

    conditional variance: constant in time
- return series – time-varying volatility, i.e. conditional variance is not constant; typical is varying of periods with extreme fluctuations and calm periods

Introduction

- typical features of return series:
  - -volatility clustering
  - -non-normal returns
  - -leverage effect
  - -comovements in volatilities
  - -non-trading periods
  - -seasonal anomalies
  - -relationship between volatility and trading volume

Introduction

- *ARCH-class models*

  -nowadays a large number of modifications of the standard ARCH model have been developed (see e.g. Bollerslev (2009): Glossary to ARCH = encyclopedic survey of ARCH-class models, downloadable)

  -ARCH-class models are widely used in macroeconomics and financial analysis

  -concerning the functional form of the conditional volatility equation - two types of models:

  linear and *non-linear*

Univariate ARCH-class models

- *linear models*, e.g.

  ARCH – Engle (1982)

  GARCH – Bollerslev (1986)

  GARCH-M – Engle, Lilien, Robins (1987)

- *non-linear models*, e.g.

  EGARCH – Nelson (1991)

  GJR – Glosten, Jagannathan a Runkle (1993)

  TGARCH – Zakoian (1990)

Multivariate ARCH-class models

- alongside the univariate ARCH-class models also multivariate volatility models (MGARCH) have been developed
- the application field of MGARCH models is broad, e.g. portfolio optimization, computation of the Value-at-Risk, analysis of the stock market co-movements, impact of crisis on stock market co-movements and assessment of the contagion effect

different types of multivariate GARCH models can be used, e.g.

VECH – Bollerslev, Engle and Wooldridge (1988)

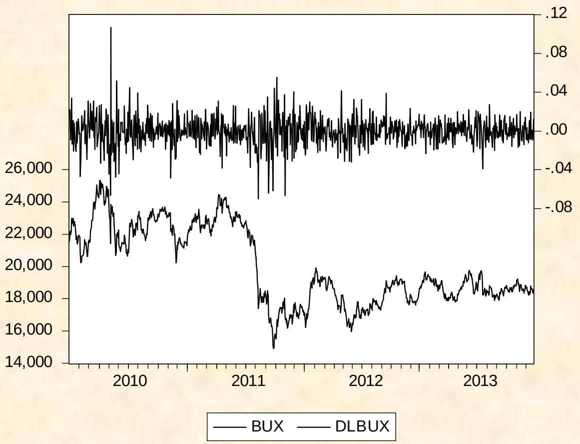
BEKK – Baba et al. (1990), Engle and Kroner (1995)

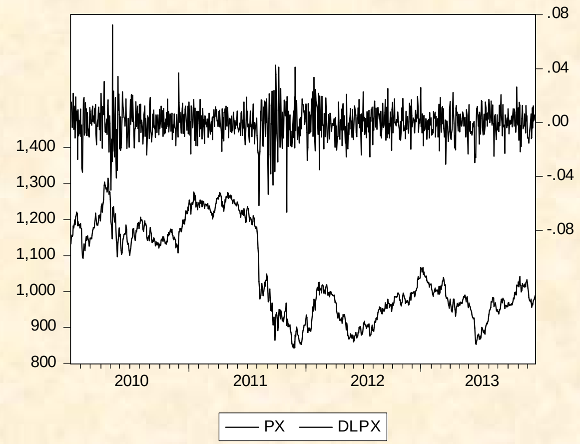
CCC – Bollerslev (1990)

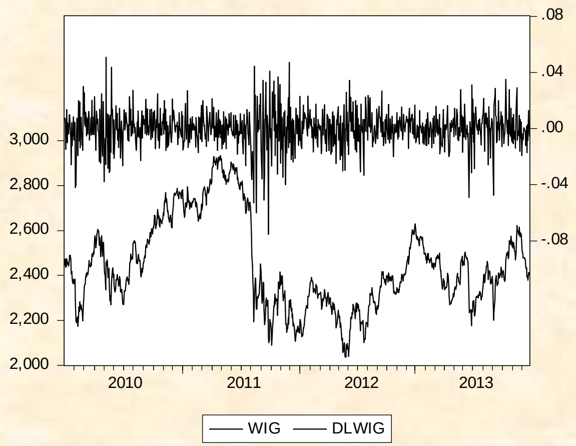
DCC – Engle (2002) and others

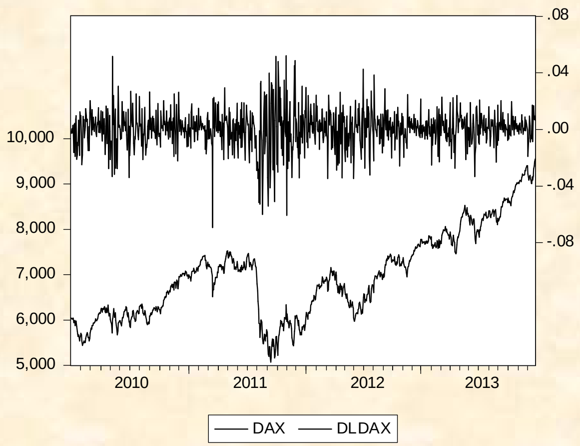
ARCH-class models and analysis of BUX, PX, WIG20 and DAX return series

- analysed period: 4.1.2010-30.12.2013
- source of data: www.stooq.com









ARCH-class models and analysis of BUX, PX, WIG20 and DAX return series

- *analysis in more steps:*
  - Descriptive statistics of logarithmic stock return series
  - Diagnostic checking (Jarque-Bera statistics, ADF statistics, Ljung-Box *Q*-statistics)
  - Specification and estimation of the conditional mean equations
  - Estimation of conditional variance equations (GARCH, GJR, EGARCH)
  - Static forecasts of logarithmic stock returns and of conditional standard deviation GARCH/GJR/EGARCH
  - Unconditional correlation coefficients
  - Estimation of DCC models

ARCH-class models and analysis of BUX, PX, WIG20 and DAX return series

- *descriptive statistics of logarithmic stock return series and some diagnostic test statistics*

	DLBUX	DLPX	DLWIG	DLDAX
Mean	-0,000156	-0,000141	$-1{,}87.10^{-5}$	0,000476
Maximum	0,106741	0,072487	0,050631	0,052104
Minimum	-0,069842	-0,066442	-0,075431	-0,069333
Std. dev.	0,014686	0,012145	0,012920	0,013508
Skewness	-0,015259	-0,347673	-0,467845	-0,387831
Kurtosis	7,944892	6,761803	6,106408	6,105617
Jarque-Bera	978,12***	585,39***	421,01***	409,86***
Diagnostic test statistics				
ADF	-31,218***	-29,381***	-30,039***	-29,442***
$Q(1)$	0,0975	2,4672	0,8504	2,4939
$Q(200)$	200,20	187,42	179,64	234,79**
$Q^2(1)$	45,444 ***	31,122***	18,856***	33,805***
$Q^2(200)$	290,12***	462,11***	633,00***	1139,5***

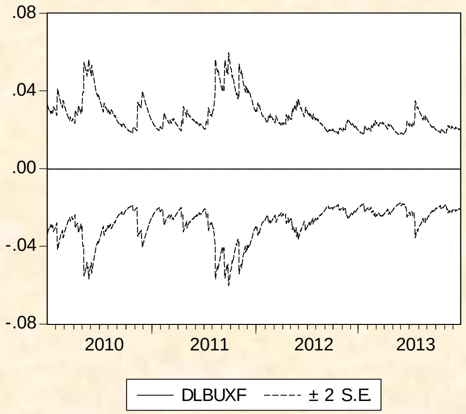
ARCH-class models and analysis of BUX, PX, WIG20 and DAX return series

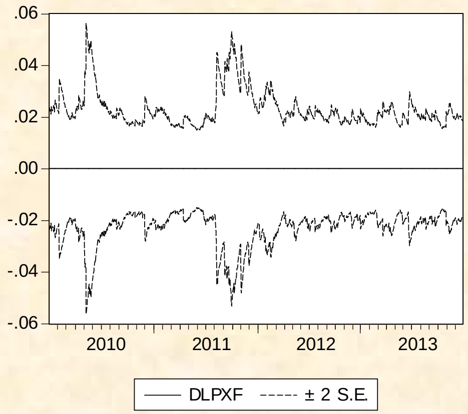
- *types of estimated univariate volatility models*

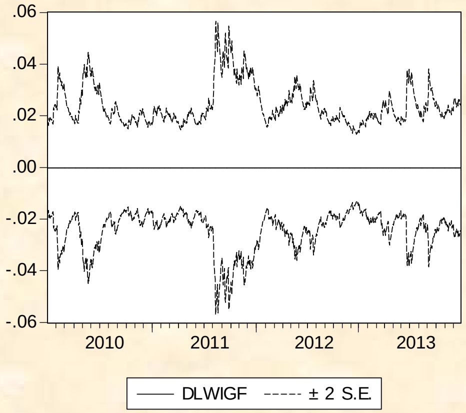
	Model type	Sign. of γ_1	Q(200)	Q^2(200)	ARCH-LM(1)	Jarque-Bera	BIC
DLBUX	GARCH(1,1)	-	205,20	211,30	0,7385	94,324***	-5,7424
	GJR(1,0,1)	Yes	206,01	217,40	0,8119	78,992***	-5,7588
	EGARCH(1,1,1)	Yes	199,93	214,56	0,5222	80,315***	-5,7487
DLPX	**GARCH(1,1)**	-	196,16	172,00	0,7189	112,110***	-6,1216
	GJR(1,1,1)	No	194,18	174,16	0,2584	98,735***	-6,1191
	EGARCH(1,1,1)	No	194,70	166,55	0,3070	95,163***	-6,1173
DLWIG	GARCH(1,1)	-	189,09	141,82	3,4132*	79,591***	-6,0045
	GJR(1,1,1)	Yes	194,63	149,74	0,5492	40,233***	-6,0345
	EGARCH(1,1,1)	Yes	197,78	156,91	0,5898	43,446***	-6,0275
DLDAX	GARCH(1,1)	-	191,64	128,24	0,0244	372,340***	-5,9561
	GJR(1,0,1)	Yes	187,89	154,85	0,0105	231,335***	-5,9987
	EGARCH(1,1,1)	Yes	187,60	152,93	0,0113	130,357***	-6,0112

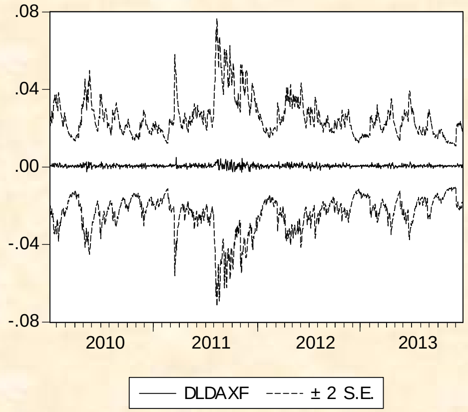
ARCH-class models and analysis of BUX, PX, WIG20 and DAX return series

- *static forecasts of logarithmic returns and +/- two standard deviations GARCH/GJR/EGARCH*









ARCH-class models and analysis of BUX, PX, WIG20 and DAX return series

- *analysis of stock market linkages based on DCC models*

  -subject of analysis: stock markets of Hungary, Czech Republic, Poland and Germany based on stock indices BUX, PX, WIG20 and DAX

  -analysis of stock market linkages:

  - high correlations between the stock returns $\Rightarrow$ rapid reduction of possible gain from international diversification
  - low correlations between returns $\Rightarrow$ markets are attractive for investors in order to exploit the potential diversification benefits

ARCH-class models and analysis of BUX, PX, WIG20 and DAX return series

- *calculation of unconditional correlation coefficients*

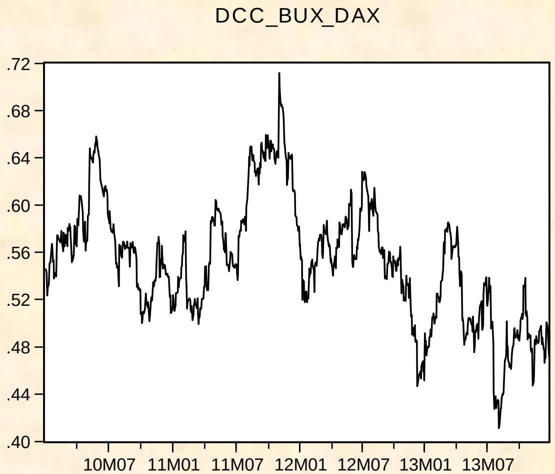
	DLBUX	DLPX	DLWIG	DLDAX
DLBUX	1,000000	0,580447	0,610067	0,608300
DLPX		1,000000	0,626299	0,630721
DLWIG			1,000000	0,685368
DLDAX				1,000000

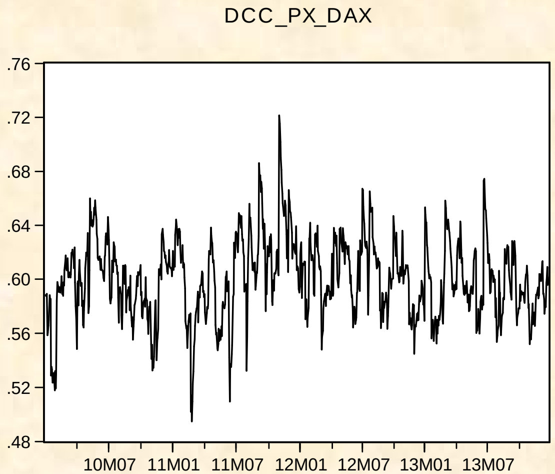
-values of unconditional correlation coefficients don’t give information about development of stock markets’ linkages in time, since it is only the single value for the whole analyzed period

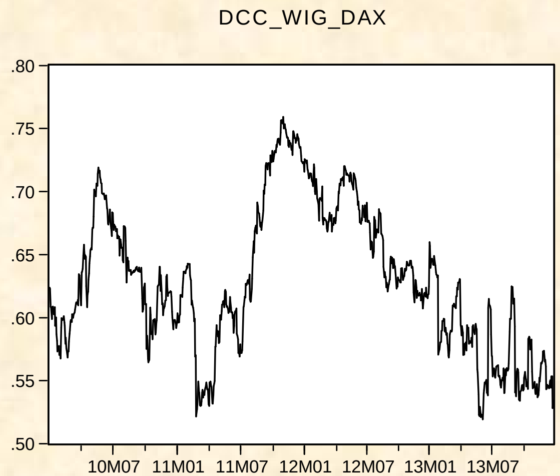
-in order to assess the development of stock markets’ linkages in time, the DCC model is beeing used

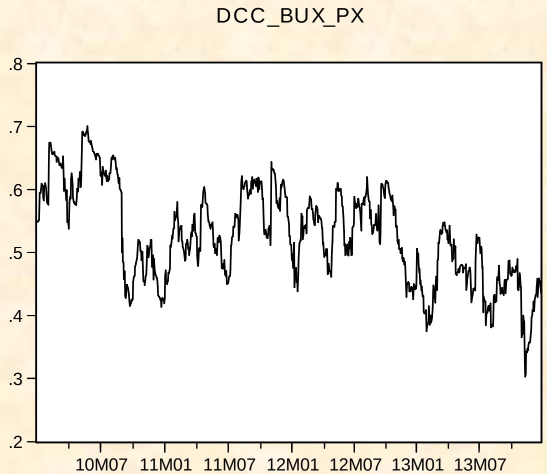
ARCH-class models and analysis of BUX, PX, WIG20 and DAX return series

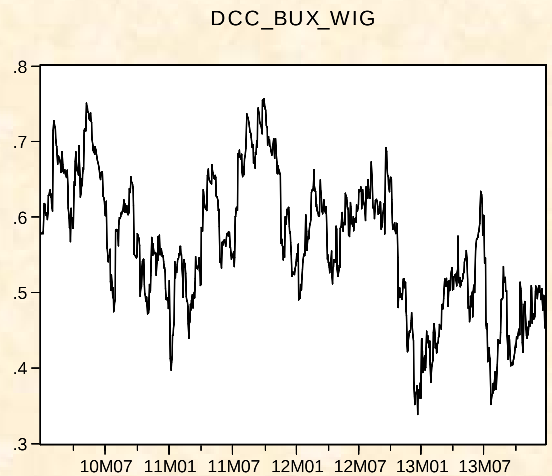
➢ *development of dynamic conditional correlations*

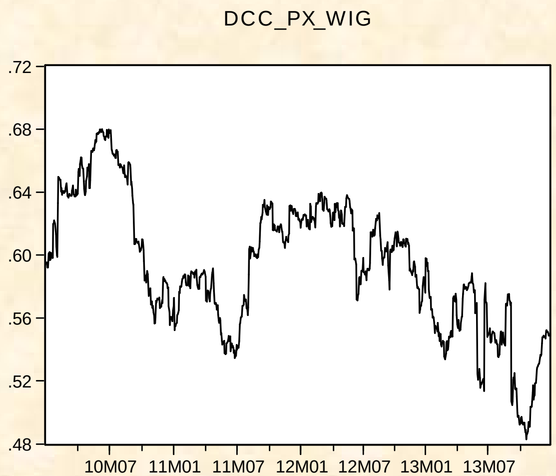












Conclusion

- despite the fact that the idea of ARCH model was published by Engle more than 30 years ago, new modifications of this model have still been published nowadays
- present state of problematics dealing with modelling of financial time series' volatility was characterized
- thereafter the presentation was concentrated on various univariate linear and non-linear ARCH-class models
- since the individual stock markets don't exist as separate markets, the presentation also included selected multivariate ARCH-class models which enable to deal with the stock market linkages

Conclusion

- in the final part of presentation the use of selected ARCH-class models was presented for analysis of Hungarian BUX, Czech PX, Polish WIG20 and German DAX
- based on DCC values (in average 0,53-0,63) we can speak about quite strong linkages of CEE markets with German stock market and also about quite strong linkages between the individual CEE stock markets

  $\Rightarrow$ these markets are not very interesting for international diversification

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