CONDITIONAL CORRELATION AND CONDITIONAL VOLATILITY

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Introduction (1)

- stock market linkages have been attracting the attention of analysts for a long time
- there exist plenty of studies dealing with this issue using various ways and methods of analysis in order to capture how shocks from one market can be transmitted to another market(s)
- it is also known that the correlation of emerging markets with the developed markets is relatively low and returns in emerging markets are much higher than in the developed markets $\Rightarrow$ opportunities for international diversification
- to analyse the co-movements of financial returns from different markets, especially to study volatility spillover, i. e. if and how the shocks from one stock market influence the volatility development of the other market is an interesting and challenging issue

Introduction (2)

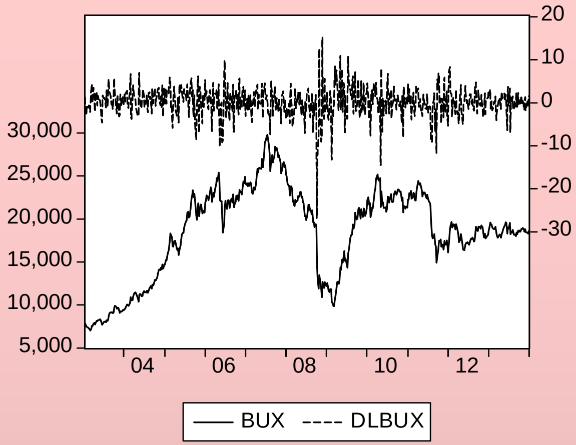
- very interesting issue is also to study the relationships between the conditional correlation and conditional volatility
- Cappiello et al. (2006) and Gjika and Horvath (2012) pointed out the fact and proved that the stock market conditional volatility and correlation are positively related
- ***Aim of the presentation***: to analyse the relationships between the dynamic conditional correlation and conditional volatility for the selected CEE markets (Czech Republic, Hungary, Poland) and the Western European stock market represented by stock market of Germany and France, respectively, as well as the relationship between German and French market and between individual CEE markets for weekly data during the period 2003-2013; the impact of the higher volatility during the period October 17th, 2008 – December 25th, 2009 is also analyzed

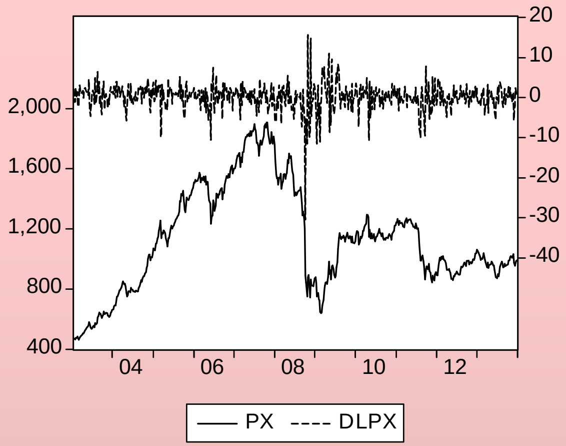
Data (1)

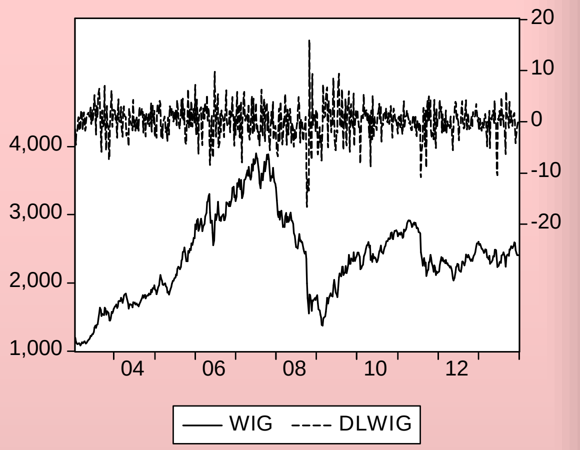
- analyzed data set: weekly data of stock price indices of CEE countries - the Czech PX, Hungarian BUX, Polish WIG20 and two Western European stock indices - the German DAX and French CAC40
- analyzed time period: January 3rd, 2003 – January 3rd, 2014
- data source: www.stooq.com
- if we assume that P_t is the closing value of the stock index at time t and r_t denotes logarithm of the corresponding stock return, the formula for calculation of the *logarithmic stock return* is as follows:

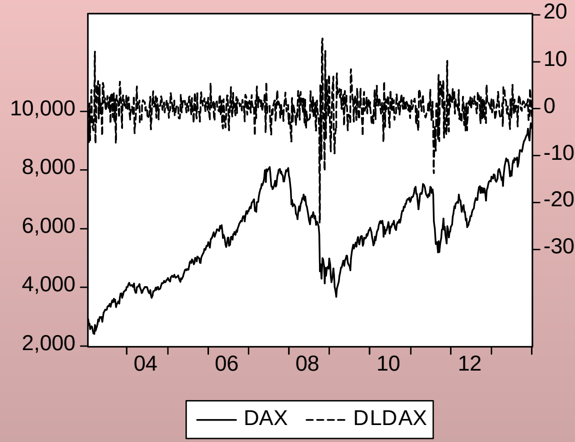
$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right).100\%$$

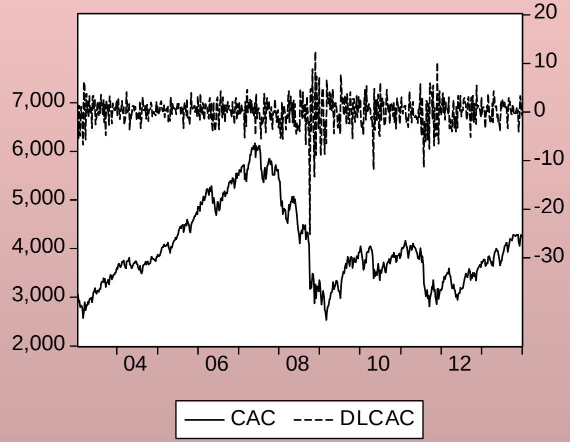
Data (2)











	DLBUX	DLPX	DLWIG	DLDAX	DLCAC
Mean	0,151860	0,132428	0,119699	0,197812	0,051615
Std. Dev.	3,562397	3,311555	3,247232	3,207019	3,043438
Skewness	-0,978616	-1,549669	-0,397702	-0,953029	-1,293369
Kurtosis	10,21027	17,57873	5,867259	10,93746	12,19645
Jarque-Bera	1332,674***	5303,720***	211,3854***	1590,943***	2178,974***

Methodological Issues (1)

- conditional mean equation

$$r_t = \omega_0 + \varepsilon_t$$

- conditional volatility – GARCH model

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}$$

- dynamic conditional correlation (DCC) – multivariate ARCH-class model
  - estimation follows in two steps:
    1) estimation of univariate ARCH-class models
    2) estimation of conditional correlation coefficients' matrix

Methodological Issues (2)

- DCC model

$k \times 1$ dimensional vector of stock returns $\mathbf{r_t}$

$$\mathbf{r_t} \big| \Omega_{t-1} \sim N(\mathbf{0}, \mathbf{H_t})$$

conditional variance-covariance matrix $\mathbf{H_t}$

$$\mathbf{H_t} = \mathbf{D_t C_t D_t}$$

$\mathbf{D_t}$ is the $\mathrm{k} \times \mathrm{k}$ diagonal matrix

with the time-varying standard deviations

from univariate GARCH models on the diagonal

$\mathbf{C_t}$ is the time-varying correlation matrix

Methodological Issues (3)

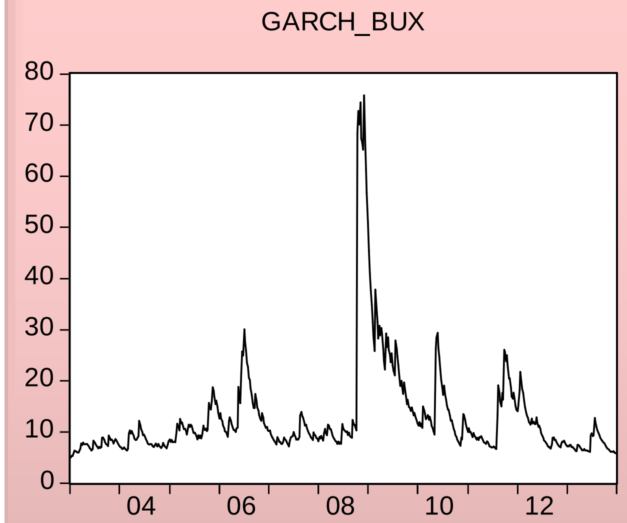
$$\mathbf{C_t} = diag(\mathbf{Q_t})^{-1/2}\mathbf{Q_t}diag(\mathbf{Q_t})^{-1/2}$$

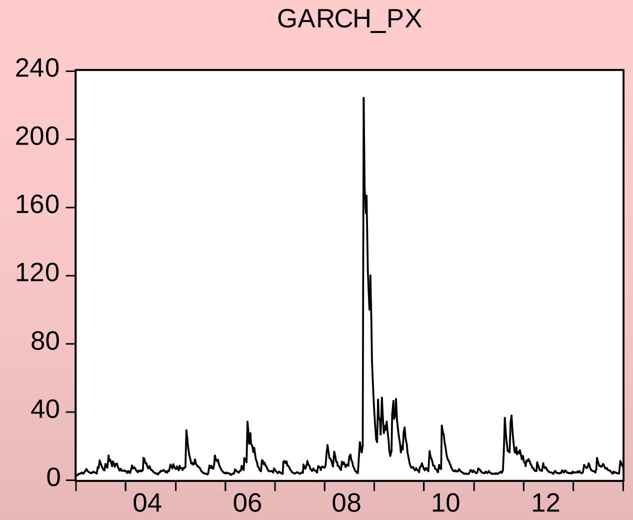
$$\mathbf{Q_t} = (1 - q_a - q_b)\mathbf{\bar{Q}} + q_a \mathbf{z_{t-1}}\mathbf{z_{t-1}^T} + q_b \mathbf{Q_{t-1}}$$

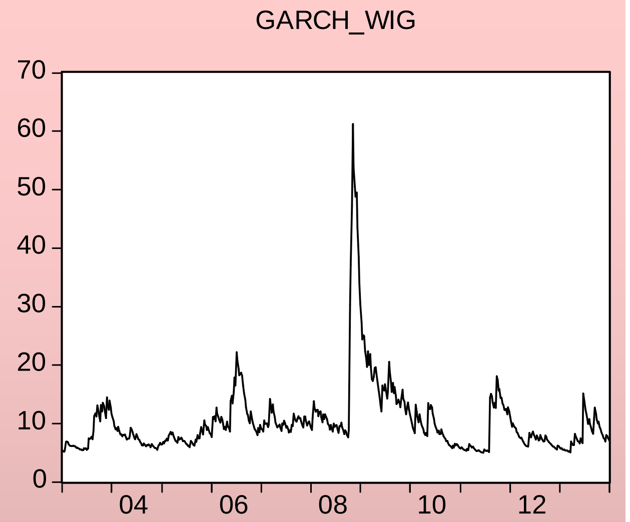
$$\mathbf{\bar{Q}} = E\left(\mathbf{z_t}\mathbf{z_t^T}\right) \qquad q_a + q_b < 1$$

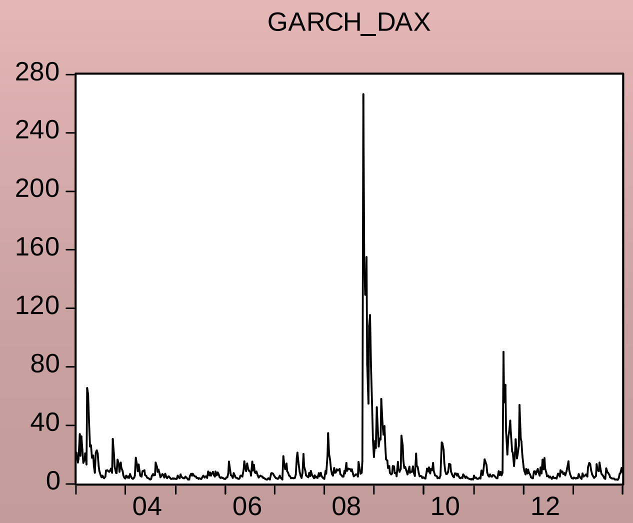
$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}\,q_{jj,t}}}; \quad i, j = 1,2,\ldots,n; \; i \neq j$$

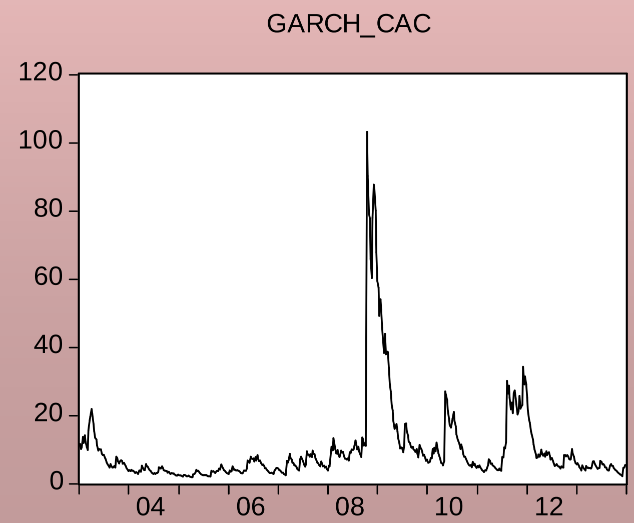
Empirical Results – Conditional Volatilities









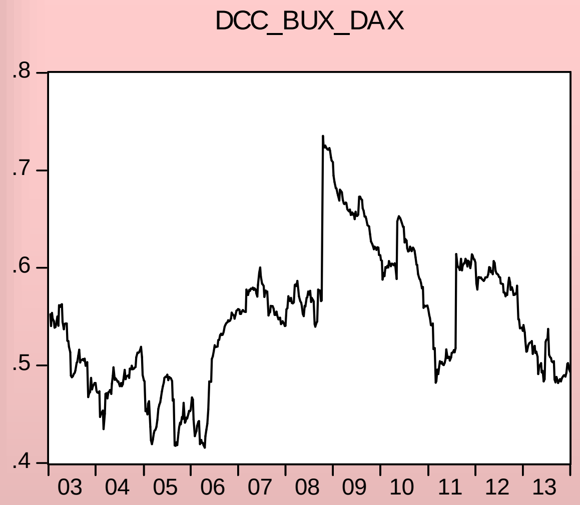


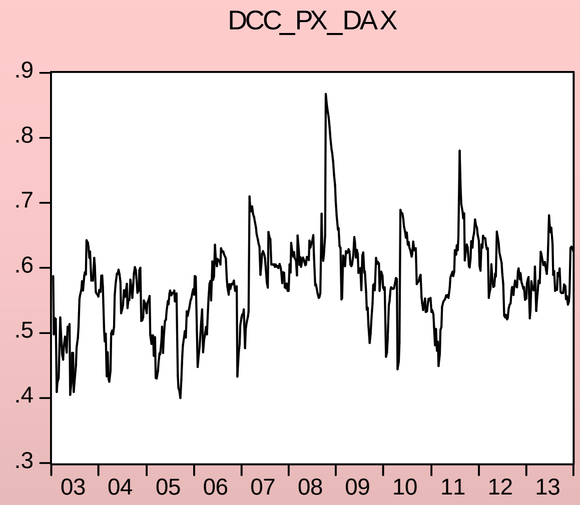
Empirical Results - Unconditional Correlations

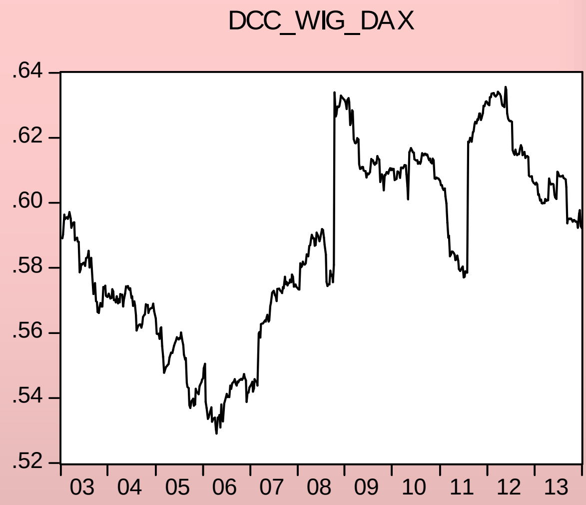
- unconditional correlations between all pairs of stock returns

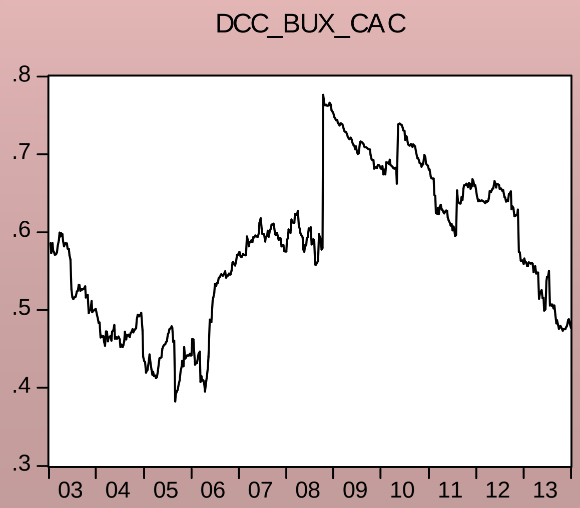
	DLBUX	DLPX	DLWIG	DLDAX	DLCAC
DLBUX	1,000000	0,714319	0,676988	0,593688	0,620141
DLPX		1,000000	0,682225	0,645627	0,676240
DLWIG			1,000000	0,609298	0,598369
DLDAX				1,000000	0,923446
DLCAC					1,000000

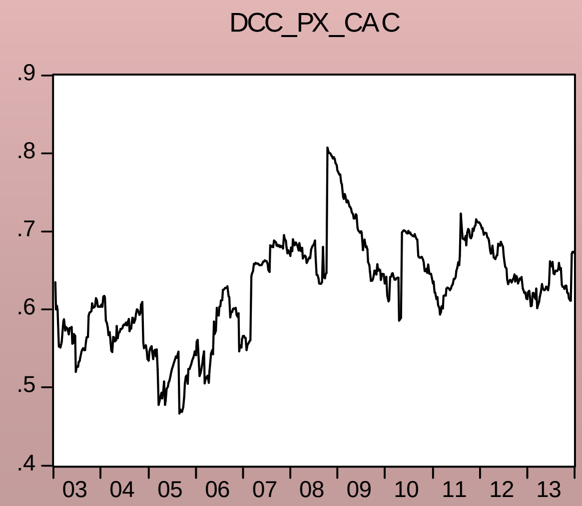
Empirical Results – Dynamic Conditional Correlations (1)

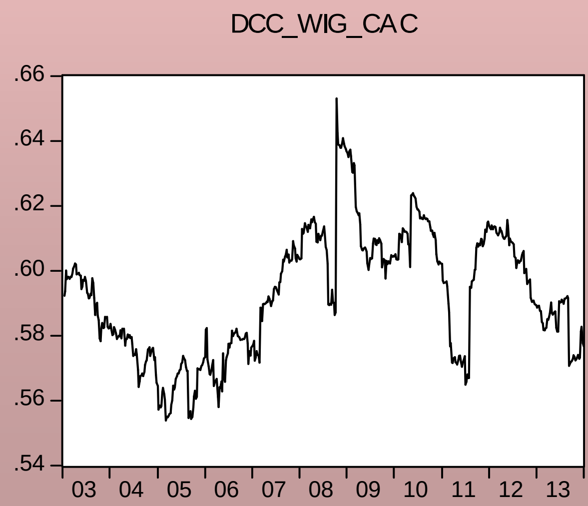




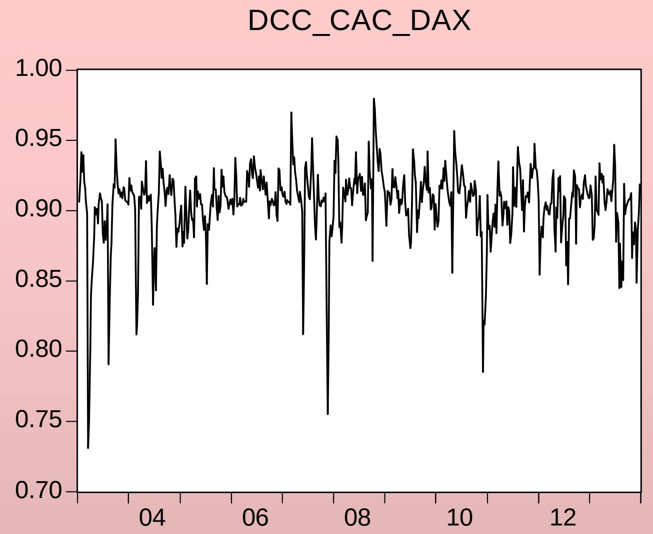


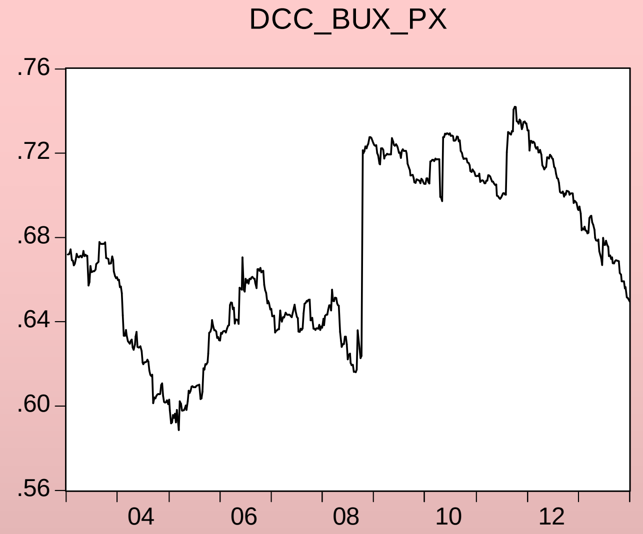


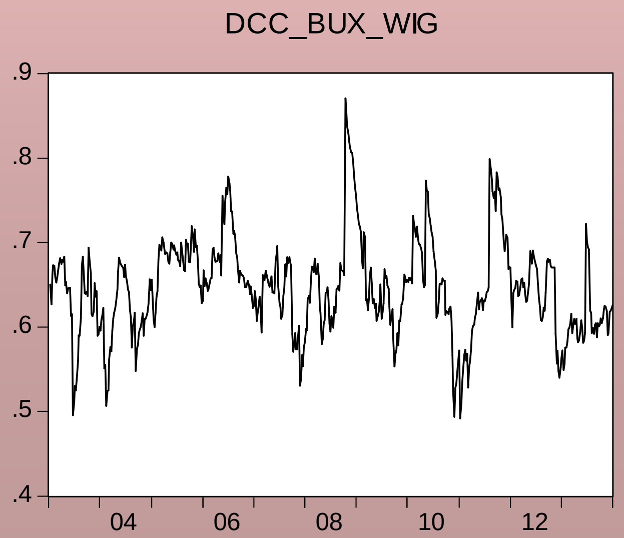


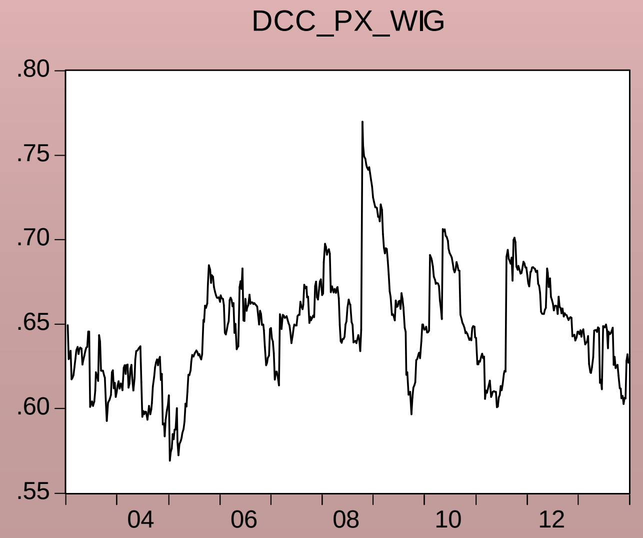


Empirical Results – Dynamic Conditional Correlations (2)









DCC and Conditional Volatility (1)

- relationship between conditional correlations and conditional volatilities

$$\rho_{ij,t} = \omega + \beta_i h_{i,t} + \beta_j h_{j,t} + \varepsilon_{ij,t}$$

- $\rho_{ij,t}$ is the estimated pair-wise conditional correlation coefficient between the stock returns of markets *i* and *j*
- positive value of β_i indicates that conditional correlations between the market *i* and *j* rise with the volatility of the market *i*, whereas negative value of β_i means that the correlations between the return series of market *i* and market *j* fall in periods of high volatility in the market *i*

DCC and Conditional Volatility (2)

- relationship between conditional correlations and conditional volatilities concerning the dummy variable dum_t taking the value of 1 in the period of higher conditional volatility (October 17th, 2008 – December 25th, 2009)

$$\rho_{ij,t} = \omega + \beta_i h_{i,t} + \beta_j h_{j,t} + \alpha_0\, dum_t + \alpha_i dum_t h_{i,t} + \alpha_j dum_t h_{j,t} + \varepsilon_{ij,t}$$

- where α_0 denotes the shift in the level of DCC during the above mentioned period, positive values of α_i and α_j indicate the rise of conditional correlations between the market *i* and *j* with the rise of volatility of the market *i* and *j*, respectively during the period October 17th, 2008 – December 25th, 2009

Empirical Results - DCC and Conditional Volatility

$$\rho_{ij,t} = \omega + \beta_i h_{i,t} + \beta_j h_{j,t} + \alpha_0\, dum_t + \alpha_i dum_t h_{i,t} + \alpha_j dum_t h_{j,t} + \varepsilon_{ij,t}$$

i	BUX	PX	WIG	CAC	BUX	PX	WIG	BUX	BUX	PX
j	DAX	DAX	DAX	DAX	CAC	CAC	CAC	PX	WIG	WIG
ω	0,486 ***	0,532 ***	0,594 ***	0,899 ***	0,505 ***	0,578 ***	0,578 ***	0,634 ***	0,543 ***	0,605 ***
β_i	0,003 ***	0,003 ***	-0,002 ***	4.10^{-4}	-0,001	-0,001 ***	-3.10^{-4}	0,004 ***	0,006 ***	0,001 ***
β_j	0,001 ***	0,001 *	0,001 ***	2.10^{-4}	0,009 ***	0,007 ***	0,002 ***	-0,001 ***	0,004 ***	0,004 ***
α_0	0,127 ***	0,028 ***	0,011	0,006	0,185 ***	0,078 ***	0,024 ***	0,075 ***	-1.10^{-5}	0,013
α_i	-0,001 *	-0,002 ***	0,003 ***	-4.10^{-4}	0,002	0,001 **	2.10^{-4}	-0,004 ***	-0,002 **	-0,001 *
α_j	-0,001 ***	-0,001 *	-0,001 ***	1.10^{-4}	-0,009 ***	-0,005 ***	-0,001 ***	0,001 **	-0,004 ***	-0,002 **

Conclusion

- concerning the statistical significance of the estimated parameters, the DCC of individual CEE return series with the Western European return series (DAX and CAC40) rise as a result of growing volatility of Western European returns (positive values of β_j), whereas during the period October 17th, 2008 – December 25th, 2009 the decline in DCC was recorded as a reaction to the rising volatility of DAX and CAC40, respectively (negative values of α_j)
- the impact of volatilities in CEE countries on DCC values was not so clear (statistically significant positive and negative values of β_i and α_i as well as in some cases non-significant parameters' values)
- no impact of volatility on conditional correlation was detected in case of DCC_DAX_CAC
- not so clear results were received for the pairs of CEE countries' returns

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