Maximizing of Portfolio Performance

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Outline

- Problem of portfolio selection
- Portfolio performance measurement techniques : Omega function, Sortino ratio
- Principle of differencial evolution (modification of algorithm to solve problem of portfolio selection)
- Computational results

Problem of portfolio selection

- Portfolio theory deals with the selection of an appropriate mix of assets in a portfolio in order to meet predetermined properties.
- The aim is to determine the allocation of the available resources in the selected group of assets that results in maximization of portfolio performance.
- The performance measurement techniques are e.g. Sharpe ratio, Treynor ratio, Jensen's alpha, Information Ratio, Sortino ratio, Omega function and the Sharpe Omega ratio etc.

Omega Function

- Omega function is measure which incorporates all the distributional, characteristics of a return series.
- The measure is a function of the returns leveled and requires no parametric assumption on the distribution.
- It considers the returns below and above a specific loss threshold and provides a ratio of total probability weighted losses and gains that fully describes the risk reward properties of the distribution

Problem of portfolio selection based on Omega function performance measure

$$\max\ \Omega(\mathbf{w}) = \frac{\sum_{t=1}^{T} \max\left(\mathbf{w}^{\mathbf{T}}\mathbf{r}_t - MAR, 0\right)}{\sum_{t=1}^{T} \max\left(MAR - \mathbf{w}^{\mathbf{T}}\mathbf{r}_t, 0\right)}$$

Subject to:

$$\mathbf{w}^{\mathbf{T}}\mathbf{e} = 1$$

$$\mathbf{w} \geq 0$$

Where:

T **represents the number of periods,**

$\mathbf{r}_t$ represents returns vector of portfolio assets in the period *t* = 1,2,…*T*,

w denotes a vector of variables w_1, w_2,…w_d representing the weight of each asset in the portfolio.

Sortino Ratio

- The Sortino ratio is based on the known Sharpe ratio, but instead of using standard deviation, the Sortino Ratio uses downside semi-variance and penalizes only those returns falling below a user-specified rate.
- This is a measurement of return deviation below a minimal acceptable rate.

Problem of portfolio selection based on Sortino ratio performance measure

$$\max\ SR(\mathbf{w}) = \frac{\mathbf{w}^{\mathbf{T}}\mathbf{E} - MAR}{\sqrt{\dfrac{1}{T} \displaystyle\sum_{\substack{t=1 \\ \mathbf{w}^{\mathbf{T}}\mathbf{r_t} < MAR}}^{T} (\mathbf{w}^{\mathbf{T}}\mathbf{r}_t - MAR)^2}}$$

subject to:

$$\mathbf{w}^{\mathbf{T}} e = 1$$

$$\mathbf{w} \geq 0$$

where
T represents the number of periods,
MAR denotes the target of required rate of return,
$\mathbf{r_t}$ denotes the return of portfolio assets in t-th period, where t =1, 2,…T,
$\mathbf{E}$ denotes the accepted returns of portfolio assets.

Differential evolution

- It belongs to the class of evolutionary techniques, that are considered to be effective tools that can be used to search for solutions of continuous non-linear problems, where is hard to use traditional mathematic methods
- The big advantage over traditional methods is that they are designed to find global extremes (with built-in stochastic component) and that their use does not require a priori knowledge of optimized function (convexity, differential etc.)

Differential evolution for solving problems of portfolio selection

If we want to apply the algorithm of differential evolution for solving problems of portfolio selection based on performance measurement, it is necessary to consider the following factors:

- Selection of an appropriate representation of individual.
- Setting of the control parameters.
- Transformation of unfeasible solutions.

Selection of an appropriate representation of individual.

The population $\mathrm{P}^{(0)}$ is randomly initialized at the beginning of evolutionary process according to the rule:

$$\mathrm{P}_{(i)}^{(0)} = w_{i,j}^{(0)} = \frac{rnd\left\langle 0,1 \right\rangle}{\sum_{j=1}^{d} w_{i,j}^{(0)}}$$

$$i = 1, 2, \ldots np \quad j = 1, 2, \ldots d$$

that ensure that the total weights of portfolio is equal to one. Each individual is then evaluated with the fitness (given by function $\Omega(\mathbf{w})$ or $SR(\mathbf{w})$).

Setting of the control parameters

d – dimensionality. Number of parameters of individual is equal to number of assets.

np – population size. Number of individuals in population. recommended setting is $5d$ to $30d$, respectively $100d$, in case the optimized function is multimodal.

g – generations. Represent the maximum number of iteration (g is also stopping criterion).

cr – crossover constant, $cr \in \langle 0,1 \rangle$. The value of cr was set on the base of statistical experiments.

f – mutation constant, $f \in \langle 0,1 \rangle$. The value of f was set on the base of statistical experiments.

Transformation of unfeasible solutions

To ensure the feasibility of solution we use the following rule: if $w_j^{test} < 0$, then $w_j^{test} = rnd\left\langle 0,1 \right\rangle$

and $\mathrm{P}_{(i)}^{test} = \mathrm{w}_{i,j}^{test} = \dfrac{w_j^{test}}{\sum_{j=1}^{d} w_j^{test}}$ $i = 1, 2, \dots np$ $j = 1, 2, \dots d$.

Empirical results

- The portfolio analyze was based on index Dow Jones Industrial, index that shows how 30 large publicly owned companies based in the United States have traded during a standard trading session in the stock market (4M Co., Alcoa Inc., American Express Co., AT&T Inc., Bank of America Corp., Boeing Co., Caterpillar Inc., Chevron Corp., Cisco Systems Inc., Coca-Cola Co., DuPont, Exxon Mobil Corp., General Electric Co., Hewlett-Packard, Home Depot Inc., IBM, Intel, Johnson & Johnson, JPMorgan Chase & Co., Kraft Foods Inc., McDonald's Corp., Merck & Co. Inc., Microsoft Corp., Pfizer Inc., Procter & Gamble Co., Travelers Cos. Inc., United Technologies Corp., Verizon Communications Inc., Wal-Mart Stores Inc., Walt Disney Co.).
- Data are processed weekly for the period July 1st 2001 to April 1st 2012. http://finance.yahoo.com/ (2012)
- A total of 559 data were analyzed.

Empirical Results

- The algorithms were implemented in MATLAB 7.1. All the experiments were run on PC INTEL(R) Core(TM) 2 CPU, E8500 @ 3.16 GHz, 3.25 GB RAM under Windows XP. The control parameters were set to: f = 0.1, cr = 0.2, np = 3000 and g = 2000.
- The input parameter of MAR (the target of required rate of return) was set to 0.055.
- Value of OMEGA FUNTION: 1.230055034 (recommended allocation is to invest assets in McDonald's companies at the rate 82.230 %, Caterpillar Inc. at the rate 13.998 % and IBM 3.772 %)
- Value of SORTINO RATIO: 0.1182 (recommended allocation is to invest assets in McDonald's companies at the rate 82.95 % and Caterpillar Inc. at the rate 17.05 %.)