Regional Economic Growth Modelling and Spatial Econometric Methodology

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Goal of the paper:

modelling of the economic growth and economic convergence based on spatial econometric instruments

- Analyses dealing with the regional growth and convergence have become very popular. Main issue of economic convergence is the question whether poor economies catch-up to wealthier economies.

- The impact of factors such as location, spillover effects from neighbouring regions or access to natural resources on the regional economic growth process can be captured and quantified by modern spatial econometric techniques.

Outline

•**Spatial econometrics** (briefly describe the issue, discuss how to incorporate the spatial dimension, different options for weighting matrices).

•**Spatial effects**: spatial autocorrelation and spatial heterogeneity (how to test for the presence of spatial dependence and how to correct for it).

Present two different spatial dependence models:

- Spatial lag model
- Spatial error model

•**Modelling of regional economic growth**

- Economic convergence models
- Economic convergence models – spatial approach

Spatial econometrics I

- The term spatial econometrics – Paelinck (1974)
- Tobler's 'First Law of Geography': „*Everything is related to everything else, but near things are more related than distant things*." Tobler (1979)
- **Motivation**: to take into account the spatial dependency, asymmetry in relationships and mutual interaction of objects and data, which are subjects of regional modelling.
- Theory of NEG (New Economic Geography) – the models of NEG enable the spatial analysis of economic data by analysing problems as regional convergence, regional concentration of economic activities and adjustment dynamics.

Spatial econometrics II

- Modelling of spatial dependence requires an appropriate representation of spatial arrangement.

Solution: relative spatial positions are represented by *spatial weights matrix* (**W**):

$$w_{ij} = 0 \text{ if } i = j$$

$$w_{ij} > 0 \text{ if } i \text{ and } j \text{ are spatially connected}$$

$$w_{ij} = 0 \text{ if } i \neq j \Rightarrow i \text{ and } j \text{ are not spatially connected}$$

$$w^{*}_{ij} = w_{ij} / \sum_{j} w_{ij}$$ is called row-standardized

- Typical forms of weights matrices:
  - Contiguity (Rook, Queen, etc.)
  - Distance (*k* nearest neighbours, Inverse distance, etc.)
  - Combination

Basic forms of spatial dependence:

- **Spatial lags** – a value of the dependent variable for a given unit is affected by variables in neighbouring units (Spatial Autoregressive Model – SAR).
- **Spatial errors** – there are some factors (not included in the model and possibly unobserved) which have an influence on all units inside an area and lead to common direction of errors of prediction of the dependent variable (Spatial Error Model – SEM).

Both types of spatial dependencies lead to problems with the classical regression model:

- Neglected **spatial lags** lead to **inconsistent** and **biased** estimates.

- Neglected **spatial errors** make model estimates **inefficient**.

The problem:

Does data exhibit spatial dependencies?

1. Detecting spatial positive or negative dependencies: e. g. Moran´s statistic *I*, Geary´s statistic *C*.
2. Detecting spatial lags or spatial errors: e.g. set of *LM* (Lagrange Multiplier) tests.

Spatial Autoregressive Model – SAR Model

Levels of the dependent variable *y* depend on the levels of *y* in neighbouring units (spillover effects):

$$\mathbf{y} = \lambda \mathbf{W}\mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$

where

- **y** - (*N* x 1) vector of a dependent variable (*N* is a size of the sample),
- **X** - (*N* x (*k*+1)) matrix of explanatory variables (*k* is a number of explanatory variables),
- **β** – ((*k*+1) x 1) vector of unknown parameters,
- **u** - (*N* x 1) vector error term,
- **W** - spatial weights matrix,
- **Wy** - spatial lag component reflecting a relationship between the dependent variable in a given unit with the same variable in neighbouring units,
- λ - parameter that represents a direction and a power of relationship between the dependent variable in a given unit with the same variable in neighbouring units and it is a subject of researcher´s interest.

Spatial Error Model – SEM Model

Spatial dependence is modelled through the error terms :

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$$

$$\mathbf{u} = \rho \mathbf{W}\mathbf{u} + \mathbf{v}$$

where

- **y** - (N x 1) vector of a dependent variable (N is a size of the sample),
- **X** - (N x k+1) matrix of explanatory variables (k is a number of explanatory variables),
- **β** - ((k+1) x 1) vector of unknown parameters,
- **W** - spatial weights matrix,
- **u** - (N x 1) vector of error term,
- **v** - assumed to be normal with $E(\mathbf{v}) = 0$ and $E(\mathbf{v}^{\mathrm{T}}\mathbf{v}) = \sigma^2\mathbf{I}$,
- ρ - parameter of spatially lagged autoregressive errors.

Model Estimation

SAR model:

- Due to the presence of *y* on both sides of the equation is not convenient to run OLS estimation, because there is a correlation between errors and regressors and the resulting estimates would be biased and inconsistent. Defined model is usually transformed to the reduced form and estimated by maximum likelihood (ML) estimation method.

SEM model:

- OLS estimates are unbiased but inefficient. Defined model is usually transformed to the reduced form and estimated by ML estimation method.

Modelling of regional economic growth

Theories of economic growth (regional) and economic convergence process (regional):

- Neoclassical theory (Solow)
- Theory of Polarization (Myrdal)
- Theory of Endogenous Growth (Romer, Perrous, Hirschman)
- Theory of New Economic Geography (Krugman, Fujita, Ciccone and Hall)

The three hypotheses concerning the regional convergence:

- ***unconditional* or *absolute*** convergence, meaning that per capita incomes converge to a common level in the long-run, if structural homogeneities (in terms of human capital, saving rate...) exist across the economies and their initial conditions do not matter,
- ***conditional* convergence**, meaning that per capita incomes converge to different levels in the long-run, if structural heterogeneities exist across the economies and their initial conditions do not matter,
- ***club* convergence**, meaning that per capita incomes converge to different levels in the long-run, if structural heterogeneities exist across the economies and their initial conditions do matter.

β-convergence

The traditional instrument for testing of convergence.

- The **absolute** β - convergence cross-country/region growth model:

$$\frac{1}{T}\ln\left(\frac{y_{i,T}}{y_{i,0}}\right) = \alpha + \beta \ln\left(y_{i,0}\right) + \varepsilon_i \qquad \varepsilon_i \sim i.i.d\left(0, \sigma_\varepsilon^2\right)$$

where

$y_{i,T}$ - per capita GDP of the region i ($i = 1,2,\ldots,n$) at time t,

T - the length of the period,

$\frac{1}{T}\ln\left(\frac{y_{i,T}}{y_{i,0}}\right)$ - the average growth rate of the i-th region per capita GDP in the period $(0, T)$,

$y_{i,0}$ - i-th region initial level of per capita GDP,

α and β - unknown parameters,

ε_i - error term.

- The absolute convergence hypothesis can be accepted if the estimated parameter β is statistically significant and negative $\Rightarrow$ not only the poor regions grow faster than the rich ones and all converge to the same level of per-capita GDP.

- Testing of the **conditional** β - convergence hypothesis is based on the following cross-sectional equation, which incorporates the variables which enable the differentiation of the regions:

$$\frac{1}{T}\ln\left(\frac{y_{i,T}}{y_{i,0}}\right) = \alpha + \beta \ln\left(y_{i,0}\right) + \boldsymbol{\gamma}\,\mathbf{X}_i + \varepsilon_i$$

where

$\mathbf{X}_i$ - vector of explanatory variables constant in the steady state equilibrium of region *i*,

$\boldsymbol{\gamma}$ - row vector of parameters.

- The conditional convergence hypothesis can be accepted if the estimated value for β is significantly negative.
- Presented models of convergence are based on the fact that each region is a geographically independent entity and no spatial interactions were considered. During the last years these models have been modified in order to incorporate the mutual regional interactions.

Spatial Autoregressive Model

- Appropriate if there exist spatial lags among neighbouring regions, i.e. if the growth rate in a region is related to those of its surrounding regions conditioning on the initial level of per capita GDP. The conditional β -convergence model can be modified as follows:

$$\frac{1}{T}\ln\left(\frac{y_{i,T}}{y_{i,0}}\right)=\alpha+\beta\ln\left(y_{i,0}\right)+\boldsymbol{\gamma}\,\mathbf{X}_i+\lambda\mathbf{W}\left\{\frac{1}{T}\ln\left(\frac{y_{i,T}}{y_{i,0}}\right)\right\}+\varepsilon_i$$

where

λ - spatial autoregressive parameter,

$\mathbf{W}$ - spatial weights matrix.

- If $\lambda \neq 0$, it means the existence of spatial effects among neighbouring regions.
- Since model contains the dependent variable on the both sides of the equation, the use of OLS will produce biased and inconsistent estimates. Therefore it has to be estimated by ML estimation method or the instrumental variables method.

Spatial Error Model

- The specification of spatial error model is appropriate when it is supposed that the spatial autocorrelation exists in the error term. In such a case the non-spatial conditional β -convergence model can be modified as follows:

$$\frac{1}{T}\ln\left(\frac{y_{i,T}}{y_{i,0}}\right)=\alpha+\beta\ln(y_{i,0})+\boldsymbol{\gamma}\,\mathbf{X}_i+\varepsilon_i$$

$$\varepsilon_i=\rho\mathbf{W}\varepsilon_i+\xi_i$$

where

ρ - spatial error parameter expressing the intensity of spatial autocorrelation between regression residuals.

- The use of OLS in case of this model would yield unbiased but inefficient estimates for the convergence and intercept parameters and biased estimate of parameter's variance. The appropriate methods of estimate are therefore ML estimation method or general methods of moments.

Conclusion

•The inclusion of spatial econometric approach into modelling of economic growth is based on the assumption that the analysed regions exist together in the same economic area and it is therefore necessary to consider these interactions also in econometric analysis.

Further research:

•to consider the geographical dimension of data in the estimation of the convergence of European regions and to emphasize geographic spillovers in regional economic growth phenomena,

•to take into account the spatial autocorrelation and construct spatial β - convergence models in order to get reliable statistical inference - an alternative way of analysing the convergence process.

It is straightforward that models using geographical data should systematically be tested for spatial autocorrelation, however not many empirical studies in the recent literature on growth theories using geographical data apply the appropriate spatial econometric tools e.g.

- BAUMONT, C. et al. 2001. *A Spatial Econometric Analysis of Geographic Spillovers and Growth for European Regions, 1980-1995*.
- FELDKIRCHER, M. 2006. *Regional Convergence within the EU-25: A Spatial Econometric Analysis*.
- PAAS, T. at al. 2007. *Econometric Analysis of income convergence in selected EU countries and their NUTS 3 level regions*.
- REY, S.J.-JANIKAS, M.V. 2005. *Regional convergence, inequality, and space*.
- REY, S.J.-MONTOURI, B.D. 1998. *US Regional Income Convergence: A Spatial Econometric Perspective*.
- VITON, P.A. 2010. *Notes on Spatial Econometric Models.*