# CVaR
# as linear programming model

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# Conditional value at risk (CVaR)

- Value at risk (VaR) a standard tool of risk management in the financial sector
- An alternative measure of risk–conditional VaR
- shortcomings of VaR:
  I. VaR measures only percentiles of profit and loss, and thus disregards the loss beyond the VaR,
  II. VaR is not a coherent risk measure because it is not sub–additive

# CVaR as linear programming model

Model of portfolio selection based on CVaR:

$$CVaR_\alpha(X) = min\left\{VaR_\alpha + \frac{1}{\alpha}E\left[\left(E_p - X - VaR_\alpha\right)^+\right]\right\}$$

where $VaR_\alpha$ – Value at risk, $E_p$ – target return, formula $(E_p - X - VaR)^+$ is a positive part of difference $E_p - X - VaR$

# CVaR as linear programming model

Variable $X$ –expectant return of portfolio $\boldsymbol{w}^T\boldsymbol{r}_k$

Objective function:

$$min\left\{VaR_\alpha+\frac{1}{\alpha t}\sum_{k=1}^{t}\left[E_p-\boldsymbol{w}^T\boldsymbol{r}_k-\mathrm{VaR}_\alpha\right]^+\right\}$$

# CVaR as linear programming model

To avoid the nonlinear formulation it is necessary to replace the element

$$\left[E_p - \boldsymbol{w}^T \boldsymbol{r}_k - VaR_\alpha\right]^+$$

with the variable $\boldsymbol{z} = (z_1, z_2, \dots, z_t)$, where $z_k \geq 0$ for $k=1,2,\dots,t$.

# CVaR as linear programming model

$$min\left\{VaR_{\alpha}+\frac{1}{\alpha t}\sum_{k=1}^{t}z_{k}\right\}$$

$$z_{k}-E_{p}+\boldsymbol{w}^{T}\boldsymbol{r}_{k}+VaR_{\alpha}\geq 0,\quad k=\{1,2,\ldots,t\},$$

$$\boldsymbol{w}^{T}\boldsymbol{E}(\boldsymbol{r}_{n})\geq E_{p}$$

$$w^{T}e=1$$

$$z\geq 0,$$

$\boldsymbol{E}(\boldsymbol{r}_{n})$ –vector of expected returns of assets