LOCATION MODEL WITH STOCHASTIC INVENTORY

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The goal of this paper is to optimize location, inventory, and allocation decisions under stochastic demand described by discrete scenarios. Objective function of proposed model minimizes the expected total cost, including location, transportation and inventory costs of distribution system over all scenarios.

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Logistics technologies and logistics operations are based on the interaction of various subsystems of the distribution system. Its optimization offers the opportunity to achieve desired results. Goal is to ensure the movement of material from point of origin to point of consumption to meet the needs of end customers.

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Decision making about distribution system design or redesign is based on given database of information. In decision making process we predict some elements of system, but we cannot predict them with certainty. In this case even deterministic models contain elements of randomness, therefore their future values can be predicted only with some probability.

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The aim of our model is to minimize the total costs of setting up, transporting, holding and ordering inventory. Therefore, our model is divided into four main parts:

- fixed installation costs,
- transportation costs,
- inventory costs,
- cost of safety stock.

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We use the following notation:

Sets

I	set of retailers, indexed by i
J	set of potential distribution centers, indexed by j
S	set of scenarios, indexed by s

Parameters

Costs

Co_j	fixed cost per year of opening DC j, for $j \epsilon J$
Ct_{ij}	per unit annual cost for transport
Cs	per unit cost for order
Ca	fixed cost for order

Demand

λ_{is}	mean annual demand at retailer i in scenario s, for $i \epsilon I, s \epsilon S$
σ_{is}^2	variance of lead time
θ_{is}^2	variance of demand at retailer i in scenario s, for $i \epsilon I, s \epsilon S$
L_j	lead time from DC to retailer
z	safety coefficient, customer service level

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Decision Variables

$$X_j = \begin{cases} 1, & \text{if is opened DC} j \in J \\ 0, & \text{otherwise} \end{cases}$$

$$Y_{ijs} = \begin{cases} 1, & \text{if retailer } i \in I \text{is served by DC} j \in J \text{in scenario } s \in S \\ 0, & otherwise \end{cases}$$

Location decisions are scenario-independent: they must be made before it is known which scenario will be realized. Assignment decisions are scenario-dependent, so the variables are indexed by scenario.

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The model is divided into four main parts:

- fixed installation costs $Co_j X_j$
- transportation costs $\sum_{i \in I} \lambda_{is} Ct_{ij} Y_{ijs}$
- inventory costs $Cs\sqrt{\frac{2Ca \sum_{i \in I} \lambda_{is} Y_{ijs}}{Cs}} + Ca \frac{\sum_{i \in I} \lambda_{is} Y_{ijs}}{\sqrt{\frac{2Ca \sum_{i \in I} \lambda_{is} Y_{ijs}}{Cs}}}$
- cost of safety stock $zCs\sqrt{\sum_{i \in I} L_j \sigma_{is}^2 Y_{ijs} + \sum_{i \in I} \lambda_{is} \theta_{is}^2 Y_{ijs}}$

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In this paper we formulate stochastic facility location model with know probability distribution for each scenario, with condition $\sum_{s\in S}\rho_s = 1$. Than objective function minimizing above mentioned costs looks like:

$$\min \sum_{s\in S}\sum_{j\in J}\rho_s \left(Co_j X_j + \sum_{i\in I}\lambda_{is} Ct_{ij} Y_{ijs} + Cs\sqrt{\frac{2Ca\sum_{i\in I}\lambda_{is}Y_{ijs}}{Cs}} + Ca\frac{\sum_{i\in I}\lambda_{is}Y_{ijs}}{\sqrt{\frac{2Ca\sum_{i\in I}\lambda_{is}Y_{ijs}}{Cs}}} + \right.$$

$$\left. +zCs\sqrt{\sum_{i\in I} L_j \sigma_{is}^2 Y_{ijs} + \sum_{i\in I}\lambda_{is}\theta_{is}^2 Y_{ijs}} \right)$$

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$$\min \sum_{s\in S} \sum_{j\in J} \rho_s \left(Co_j X_j + \sum_{i\in I} \lambda_{is} Ct_{ij} Y_{ijs} + Cs\sqrt{\frac{2Ca\sum_{i\in I}\lambda_{is}Y_{ijs}}{Cs}} + Ca\,\frac{\sum_{i\in I}\lambda_{is}Y_{ijs}}{\sqrt{\frac{2Ca\sum_{i\in I}\lambda_{is}Y_{ijs}}{Cs}}} + \right.$$

$$\left. +zCs\sqrt{\sum_{i\in I} L_j \sigma_{is}^2 Y_{ijs} + \sum_{i\in I} \lambda_{is}\theta_{is}^2 Y_{ijs}} \right)$$

subject to

$$\sum_{j\in J} Y_{ijs} = 1 \qquad i\in I, s\in S$$

$$Y_{ijs} \le X_j \qquad i\in I, j\in J, s\in S$$

$$X_j \in \{0,1\} \qquad j\in J$$

$$Y_{ijs} \in \{0,1\} \qquad i\in I, j\in J, s\in S$$