Econometric Modelling of Financial Time Series: Selected Approaches

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The main aim of the presentation

- a brief introduction into the econometric modelling of the financial time series

2 parts:

- *volatility modelling using the ARCH-class models*
- *investigation of the relationships between pairs of time series based on correlation analysis, cointegration concept and Granger causality with illustration for the Netherland stock exchange data*

Introduction (ARCH-class Models)

- financial time series are mostly analysed in the form of returns (first differences), the main feature of which is the time-varying volatility
- conditional volatility approach - Engle (1982), Bollerslev (1986), etc.
- today - huge amount of various ARCH-class models
- from the perspective of the rational investor represents the height of returns only one part of the decision making process
- another essential factor to take into account is the amount of risk, i.e. the volatility of returns together with its seasonal anomalies (turn-of-the-year, turn-of-the-month, day-of-the-week effects, etc.)
- interesting area is also to investigate the role of the trading volume in explanation of the volatility persistence of stock returns

Data and Methodology

- the analysis is beeing usualy done on logarithmic transformation of return series
- the logarithmic return series are also calculated as the logarithmic first differences of the individual series measured in levels, e.g. daily closing values of the stock indices or daily exchange rates:

$$r_t = d(\ln(P_t)) = \ln\left(\frac{P_t}{P_{t-1}}\right)$$

where P_t is the „price“ (e.g. closing value of the stock index, exchange rate) at time t and r_t denotes logarithm of the corresponding return

Methodological aspects:

- the logarithmic returns equation, i.e. the conditional mean equation - Box-Jenkins ARMA(m,n) model
- conditional variance equation - appropriate ARCH-class model

Some ARCH-class Models

Model/ authors, year	Mathematical form/ parameter conditons/some explanations
ARCH(q) Engle, 1982	$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \ldots + \alpha_q \varepsilon_{t-q}^2$ $\alpha_0 > 0$, $\alpha_i \geq 0$ for $i = 1,2,\ldots,q$
GARCH(p,q) Bollerslev, 1986	$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}$ $\alpha_0 \geq 0, \alpha_i \geq 0$ for $i = 1,2,\ldots,q$, $\beta_j \geq 0$ for $j = 1,2,\ldots,p$
IGARCH(p,q) Engle and Bollerslev, 1986	$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j}$ for $\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j = 1$
EGARCH(p,q,r) Nelson, 1991	$\ln(h_t) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \frac{\lvert\varepsilon_{t-i}\rvert}{\sqrt{h_{t-i}}} + \sum_{j=1}^{p} \beta_j \ln(h_{t-j}) + \sum_{k=1}^{r} \gamma_k \frac{\varepsilon_{t-k}}{\sqrt{h_{t-k}}}$ leverage effect: $\gamma_k < 0$, asymmetry in volatility: $\gamma_k \neq 0$
TGARCH(p,q) Zakoian, 1990	$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} + \sum_{i=1}^{q} \gamma_i \varepsilon_{t-i}^2 I_{t-i}^-$ $\alpha_0 > 0$, $\alpha_i \geq 0$, $\alpha_i + \gamma_i \geq 0$ for $i = 1,2,\ldots,q$, $\beta_j \geq 0$ for $j = 1,2,\ldots,p$ $I_{t-i}^- = \begin{cases} 1, if\ \varepsilon_{t-i} < 0 \\ 0, if\ \varepsilon_{t-i} > 0 \end{cases}$, leverage effect: $\gamma_i > 0$, asymmetry in volatility: $\gamma_i \neq 0$
PARCH(p,q) Ding et al., 1993	$\left(\sqrt{h_t}\right)^\delta = \alpha_0 + \sum_{i=1}^{q} \alpha_i \left(\lvert\varepsilon_{t-i}\rvert - \gamma_i \varepsilon_{t-i}\right)^\delta + \sum_{j=1}^{p} \beta_j \left(\sqrt{h_{t-j}}\right)^\delta$ $\alpha_0 > 0, \delta \geq 0, \beta_j \geq 0$ for $j = 1,2,\ldots,p$, $\alpha_i \geq 0$ a $-1 < \gamma_i < 1$ for $i = 1,2,\ldots,q$, asymmetry in volatility: $\gamma_i \neq 0$

Modelling of seasonal anomalies: a GARCH(p,q) case study

- Conditional mean equation

$$r_t = \omega_0 + \sum_{j=1}^{m} \phi_j r_{t-j} + \varepsilon_t + \sum_{k=1}^{n} \theta_k \varepsilon_{t-k} + \sum_{l=2}^{12} \mathrm{K}_l Y_{lt} + \Lambda M_t + \sum_{r=2}^{5} \Pi_r D_{rt}$$

- Conditional variance equation

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} + \sum_{l=2}^{12} \kappa_l Y_{lt} + \lambda M_t + \sum_{r=2}^{5} \pi_r D_{rt}$$

turn-of-the-year effect: dummy variables Y_{lt} $(l = 2,3,\ldots,12)$ representing months from February to December

turn-of-the-month effect: dummy variable M_t taking the value of one in the month's first fifteen calendar days (and zero in the remaining days)

day-of-the-week effect: dummy variables D_{rt} $(r = 2,3,4,5)$ for individual trading days with exception of Monday

The role of the trading volume in explanation of the volatility persistence of stock returns

GARCH(p,q) case study

- to examine the effect of trading volume on stock returns volatility, the following modification of the conditional variance equation is used

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} + \delta V_t$$

  where V_t is the logarithm of the trading volume

- according to the Lamoureux et al. (1990) the parameter δ should be positive and the volatility persistence should become negligible

Interactions between the Financial Time Series

- e.g. mutual relationships between:
  - :exchange rates and stock indices
  - :pairs of exchange rates
  - :pairs of stock indices
- various procedures:
  - :graphical analysis
  - :correlation procedures
  - :long-run relationships - cointegration procedures (Engle-Granger, Johansen)
  - :short-run relationships - Granger causality, impulse responses analysis

Long-run relationships

- cointegration procedures:
  - Engle-Granger = cointegration approach for a bivariate system (one cointegrating vector)

    -based on testing of residuals from cointegrating equation performing e.g. the ADF test on residuals
  - Johansen = more general technique applicable also in case of more than two variables (for *N* number of variables we can have up to *N-1* cointegrating vectors)

    -based on maximum likelihood method using both the λ_{trace} and $\lambda_{\max}$ statistics

Short-run relationships

Granger causality

-we can say, that the time series x_t Granger-causes time series y_t, if y_t can be predicted better by using past values of x_t than by using only the historical values of the y_t

-if this doesn't hold, we can say that x_t doesn't Granger-cause y_t

-whether the time series y_t Granger-causes the time series x_t can be tested in analogical way

-the corresponding VAR(k) model:

$$y_t = \alpha_{10} + \sum_{j=1}^{k} \alpha_{1j} x_{t-j} + \sum_{j=1}^{k} \beta_{1j} y_{t-j} + \varepsilon_{1t}$$

$$x_t = \alpha_{20} + \sum_{j=1}^{k} \alpha_{2j} x_{t-j} + \sum_{j=1}^{k} \beta_{2j} y_{t-j} + \varepsilon_{2t}$$

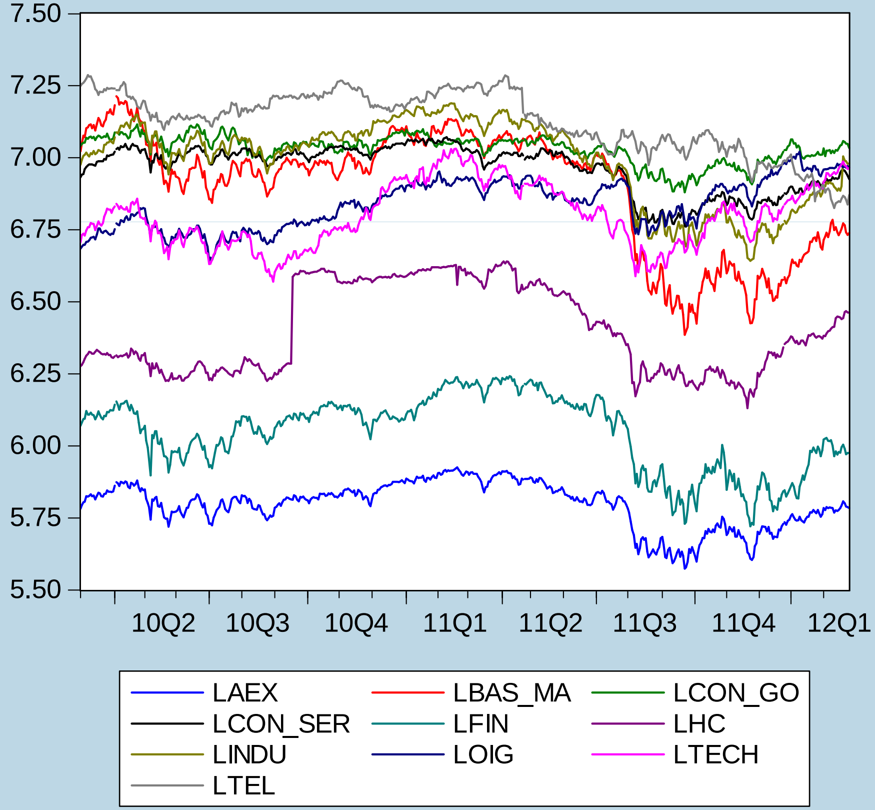
-testing statistics: Wald F-statistics

Impulse responses analysis

-to examine the short-run dynamic relations between analysed time series

Relationships between the overall stock market index and its sectoral indices: A Case Study for Netherland

Graphical analysis



general index: AEX

sectoral indices:

Basic Materials (BAS_MA),
Consumer Goods (CON_GO),
Consumer Services (CON_SER),
Financials (FIN), Health Care (HC),
Industrials (INDU), Oil&Gas (OIG),
Telecomunications (TEL),
Technology (TECH)

source of data: NYSE Euronext webpage

Note: prefix „L“ denotes natural logarithm

Descriptive statistics and ADF test results - indices

Indices	LAEX	LBAS_ MA	LCON_ GO	LCON_ SER	LFIN	LHC	LINDU	LOIG	LTECH	LTEL
Mean	5,7966	6,8978	7,0309	6,9644	6,0459	6,4040	6,9918	6,8397	6,8011	7,1264
Maximum	5,9248	7,2135	7,1161	7,0718	6,2426	6,6398	7,1894	7,0207	7,0308	7,2864
Minimum	5,5738	6,3854	6,8797	6,7609	5,7209	6,1313	6,6417	6,6425	6,5696	6,8262
Std. Dev.	0,0793	0,2063	0,0481	0,0809	0,1282	0,1519	0,1430	0,0840	0,1080	0,1078
Skewness	-0,7260	-0,8168	-0,9615	-0,9383	-0,5488	0,2199	-0,7487	-0,0967	0,1317	-0,7263
Kurtosis	2,8608	2,3760	3,4727	2,6388	2,3292	1,4261	2,3029	1,9028	2,0779	2,7439
Jarque-Bera	45,7***	65,6***	84,1***	78,4***	35,5***	57,3***	58,5***	26,6***	19,7***	46,7***
ADF stat. (without trend and without constant)	-0,0207	-0,6035	-0,0463	-0,0442	-0,2334	0,3981	-0,0146	0,8111	0,6651	-1,3302

*Note: Symbols ***, **, * denote in the whole presentation the rejection of the null hypothesis on the 1 %, 5 %, and 10 % significance level, respectively.*

Descriptive statistics and ADF test results - returns

Returns	DLAEX	DLBAS _MA	DLCON _GO	DLCON_ SER	DLFIN	DLHC	DLINDU	DLOIG	DLTECH	DLTEL
Mean	0,0000	-0,0006	0,0000	0,0000	-0,0002	0,0004	0,0000	0,0005	0,0005	-0,0007
Maximum	0,0707	0,0852	0,0607	0,0396	0,1410	0,3021	0,1031	0,0400	0,0548	0,0366
Minimum	-0,0457	-0,0840	-0,0393	-0,0463	-0,0886	-0,0997	-0,0760	-0,0493	-0,0536	-0,0862
Std. Dev.	0,0134	0,0219	0,0115	0,0100	0,0215	0,0189	0,0165	0,0125	0,0165	0,0126
Skewness	0,0207	0,0217	0,0944	-0,3599	0,3247	7,5419	0,1479	-0,4060	-0,0161	-1,0377
Kurtosis	5,4520	5,0422	4,8235	5,2544	7,6347	130,1668	7,6306	4,7117	3,8564	9,2332
Jarque-Bera	128,8***	89,4***	72,0***	119,9***	469,1***	351210,0 ***	461,1***	76,9***	15,7***	924,3***
ADF stat. (trend+constant)	-21,1106 ***	-21,2356 ***	-22,3070 ***	-20,3623 ***	-21,6441 ***	-22,3557 ***	-20,3662 ***	-19,6799 ***	-21,6954 ***	-21,1670 ***

Correlation coefficients of AEX index with its sectoral indices

- based on the graphical analysis we can expect that there are some relationships between the overall index (AEX) and some of its sectoral indices which was proved by calculation of correlation coefficients:

Correl. coeff.	LBAS_ MA	LCON_ GO	LCON_ SER	LFIN	LHC	LINDU	LOIG	LTECH	LTEL
levels	0,9244	0,8484	0,9171	0,9393	0,7535	0,9440	0,2013	0,6142	0,6516
returns	0,8948	0,8488	0,8446	0,9249	0,3948	0,8803	0,7876	0,7601	0,5056

Engle-Granger cointegration test – residuals ADF test statistics

	LBAS_MA	LCON_GO	LCON_SER	LFIN	LHC	LINDU	LOIG	LTECH	LTEL
ADF stat.	-2,48018	-3,02735	-2,33871	-2,08952	-2,45456	-2,75117	-1,78739	-0,87917	-0,78831

➢it was not possible to reject the unit root hypothesis, i.e. no long-run relationship was confirmed

Granger causality test – results

H0: AEX index returns does not Granger cause sectoral index returns

	F-stastistics				
	Lags: 1	Lags: 2	Lags: 3	Lags: 4	Lags: 5
DLBAS_MA	0,2579	2,6568*	2,3116*	2,2241*	1,7889
DLCON_GO	0,2134	0,7560	0,8691	0,6923	0,6442
DLCON_SER	9,3063***	4,8284***	3,2660**	2,5504**	2,1158*
DLFIN	1,2918	0,6601	0,5325	0,5528	0,5566
DLHC	2,5155	1,1845	0,8099	0,6098	0,5093
DLINDU	3,2203*	3,4277**	2,5070*	2,2781*	3,2654***
DLOIG	0,0686	0,0370	0,3193	0,3748	0,5706
DLTECH	0,1174	0,2553	1,3916	1,0361	1,4629
DLTEL	0,0332	0,8665	1,7047	1,4140	1,1833

A Case Study Conclusion

- it was not possible to reject the hypothesis about the existence of the unit root for the overall AEX index as well as for all the sectoral indices; the returns were stationary
- concerning the graphical analysis and correlation coefficients, the strongest relationships (correlation coefficient more than 0,9) were indentified between LAEX and following sectoral indices: LBAS_MA, LCON_SER, LFIN, LINDU
- based on Granger causality test results, it was proved that for pairs with confirmed short-run relationships the correlation coefficients were high
- no long-run relationship was confirmed
- similarly as Waściński et al. (2009) we proved the existence of the short-run relationships in some cases (although the results are strongly dependent on the concrete analysed pair of indices), but the long-run relationships of the overall AEX index to its sectoral indices were confirmed in neither case

References

- ***Bollerslev T.*** (1986) Generalized Autoregressive Conditional Heteroscedasticity. Journal of Econometrics, Vol. 31, No.3.
- ***Chocholatá M.*** (2012) Relationships Between General Index and its Sectoral Indices: A Case Study for Selected European Indices. Proceedings of the International Scientific Conference 30th May – 1st June 2012, Bratislava, Slovakia, pp. 78 – 84.
- ***Engle R.F.*** (1982) Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation, Econometrica, Vol. 50, No. 4.
- ***Engle R. F., Granger C. W. J.*** (1987) Co-integration and Error Correction: Representation, Estimation, and Testing. Econometrica, Vol. 55, pp. 251 – 276.
- ***Granger C. W. J.*** (1969) Investigating Causal Relations by Econometric Models and Cross-Spectral Methods. Econometrica, Vol. 37, July, pp. 424 - 438.
- ***Lamoureux C., Lastrapes N.*** (1990) Heteroscedasticity in stock return data: volume versus GARCH Effects, The Journal of Finance, Vol. XLV, No. 1, pp. 221-229.
- ***Johansen S.*** (1988) Statistical Analysis of Cointegration Vectors. Journal of Economic Dynamics and Control, Vol. 12, pp. 231-254.
- ***Waściński T., Przekota G., Sobczak, L.*** (2009) A Long- and Short-Term Relationship Between Sectoralindexes and the WIG (Warsaw Exchange Index). Polish Journal of Environmental Studies, Vol. 18, No. 5B, pp. 237-242.