Heuristic Solution of the Vehicle Routing Problem

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The vehicle routing problem (VRP)

- to design the optimal set of routes for a vehicle in order to serve a given set of customers
- the vehicle has a certain capacity and it is located in a certain depot
- there exists a matrix that represents the minimum distance (length, cost, time) between all the pairs of the customers and also between the customers and the depot

The vehicle routing problem (VRP)

- the goal: to find optimal vehicle routes (usually minimum distance).
- the routes must be designed in such a way that each point is visited only once by exactly one vehicle, all routes start and end at the depot, and the total demands of all points on one particular route must not exceed the capacity of the vehicle.

EVOLUTIONARY ALGORITHMS

- nontraditional computing techniques whose common characteristic is that they are inspired by the observation of the nature processes (genetic algorithms, ant colony optimization, differential evolution)
- the big advantage over traditional methods is that they are designed to find global extremes (with built-in stochastic component)
- they involve a search from a "population" of individuals, not from a single one and the searching process is based in cooperation of individuals.

SELF ORGANIZING MIGRATING ALGORITHM SOMA

- can be classified as evolutionary algorithm, because it is based on the self organizing behavior of individuals in a social environment (e.g. a herd of animals looking for food) despite the fact that no new individuals are created during the simulation.
- can be also classified as memetic algorithm, where the basic characteristic is the use of competitive cooperative strategies with synergic attributes.

Principle of SOMA

- Individual – set of arguments of objective funcion (*d* – number of parameters of individual), each individual represents a candidate solution of a given problem
- Population – set of individuals (*np*–number of individuals in population)
- Fitness – value of objective function for each individual

	$f_c(\mathbf{x}_i)$	1	2	....	*d*
$\mathbf{x}_1$	$f_c(\mathbf{x}_1)$	x_{11}	x_{12}		x_{1d}
$\mathbf{x}_2$	$f_c(\mathbf{x}_2)$	x_{21}	x_{22}		x_{2d}
. . .					. . .
$\mathbf{x}_{np}$	$f_c(\mathbf{x}_{np})$	x_{np1}	x_{np2}	…….	x_{npd}

Principle of SOMA

- Leader: max $f_c(x_i)$
- Each individual is moving towards the Leader

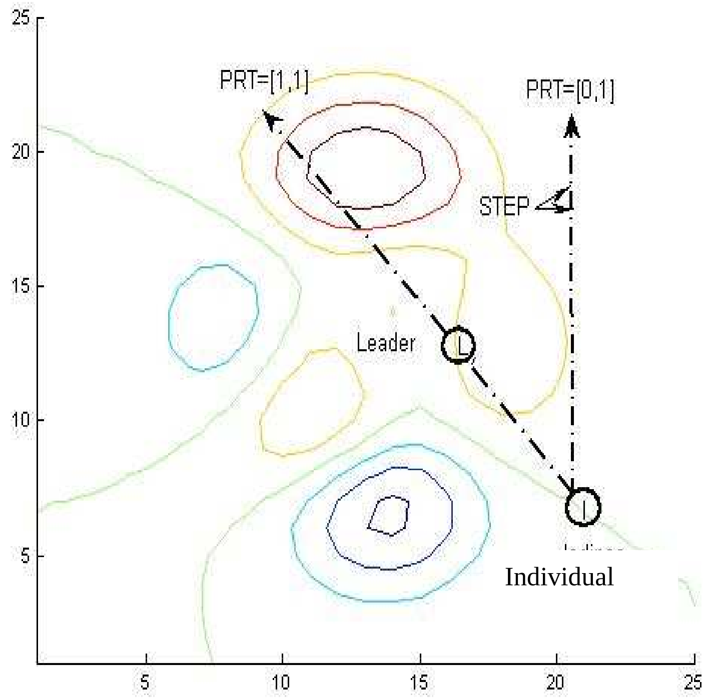
$$x_{i,j}^{mk+1} = x_{i,j,start}^{mk} + (x_{L,j}^{mk} - x_{i,j,start}^{mk})tprt$$

$$t \in \langle 0, \text{by } step \text{ to } mass \rangle$$

- Individual returns to the positio with the best fitness:

$$x_{i,j}^{mk+1} = \min\left\{ f_c(x_{i,j}^{mk}), f_c(x_{i,j,start}^{mk} \right\}$$

$$\text{PRT}_j = \begin{cases} 1, \text{ if } rand_j \langle 0,1 \rangle > prt \\ 0, \text{ otherwise} \end{cases}$$



PSEUDOCODE

BEGIN

SETTING of control parameters;

INITALIZATION of population;

EVALUATION of each individual;

WHILE (*STOPPING CRITERION* is not satisfied) **DO** **(***MIGRATION LOOPS***)**

SELECT leader

FOR (each individual of the population except leader) **DO**

JUMPING individual toward the leader

EVALUATE fitness of individual after each jump

MOVE individual on the position with the best fitness

ENDFOR

ENDWHILE

EVALUATE process of calculating

SOMA for VRP

- Selection of an appropriate representation of individual
- Formulation of objective function
- Transformation the parameters of individual to the real numbers
- Transformation of unfeasible solutions
- Setting of the control parameters of SOMA

Selection of an appropriate representation of individual

- natural representation of individual (each node (city) except center node (depot) is assigned with integer from 1 to *n* (*n* represents the number of nodes except depot), which represents corresponding node in individual).
- each individual is then represented by *n*-dimensional vector of integers, representing the sequence of visiting of the nodes.
- Fitness: total cost (total distance) of the routes

Formulation of objective function

- The sum of all demands on the route must not exceed the capacity of vehicle
- The supply nodes are included in the same route only in case of non negative value of savings (based on heuristic Clarke-Wright algorithm)

Transformation the parameters of individual to the real numbers

- Let z_i represents an integer and r_i represents real number, than:

$$r_i = -1 + \frac{z_i * f * 5}{10^3 - 1}$$

$$z_i = \frac{(1 + r_i) * (10^3 - 1)}{5 * f}$$

$$\alpha_i = \text{int}(z_i + 0{,}5)$$

$$\beta_i = \alpha_i - z_i$$

Transformation of unfeasible solutions

a) Parameter of individual after the transformation from real numbers to integers is less then 1 or greater then *d*, in this case the relevant parameter is replaced by new randomly generated parameters in range 1 to *d*.

b) Created individual does not comprise a permutation of integers *d. C*orrection approach:

1. Let **m** is the vector of parameters of the individual dimension of *d* with *k* different elements. If *d* - *k* = 0, go to step 4). Otherwise, go to step 2).
2. Create the vector **p** (dimension *d* - *k*) of random permutation of such *d* - *k* elements,
   which are not included in the vector **m**. If the number of non-zero components of the vector **p** = 0, go to step 4). Otherwise, find the first repeated element of vector **m**. Let this element be *mc* and let the first nonzero element of vector **p** be *pk* . Set *mc* = *pk* and go to step 3).
3. Set *pk* = 0 and return to the step 2).
4. Return **m**

Experiments

Problem	Eil23	Eil 30
Simulation 1	569	548
Simulation 2	570	548
Simulation 3	569	548
Simulation 4	569	539
Simulation 5	570	534
Simulation 6	600	535
Simulation 7	570	589
Simulation 8	570	537
Simulation 9	572	562
Simulation 10	576	589
Optimum	569	534
Minimum	569	534
Mean	573,5	552,9
Perc. deviation	0,79%	3,54%