THE MAINTENANCE OF A MINIMUM NUMBER OF ROAD TRAFFIC

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Minimum spanning tree problem

- Assume connected weighted graph with n nodes and edges.
- Every edge is evaluated with weight c_{ij} (i = 1, 2,...n, j = 1, 2,...n).
- The goal is to find minimum spanning tree so that the sum of used edges is minimal (minimal lenght of used route).

Problem formulation

- binary variables x_{ij}, for i, j =1, 2,...n

  The variable will be equal to 1, if the edge between *i*-th a *j*-th node is a part of system of used roads

  The variable is equal to 0 otherwise.

- variables y_{ij}, for i, j =1, 2,...n, are limited by interval $\langle 0;(n-1)\rangle$

  Represent the number of using of relevant route and also they preserve there is a connected route from relevant node to the initial node.

Problem formulation

$$f\left(\mathbf{x},\mathbf{y}\right)=\sum_{i=1}^{n}\sum_{j=1}^{n}c_{ij}x_{ij}\rightarrow \min$$

$$\sum_{j=2}^{n}x_{1j}=0$$

$$\sum_{j=1}^{n}x_{ij}=1,\quad i=2,3,\ldots n$$

$$\sum_{j=1}^{n}y_{ij}-\sum_{j=2}^{n}y_{ji}=1,\qquad i=2,3,\ldots n-1$$

$$0\le y_{ij}\le\left(n-1\right)x_{ij},\qquad i,j=1,2,\ldots n$$

$$x_{ij}\in\left\{0,1\right\},\qquad i,j=1,2,\ldots n$$

Multicriteria programming problem

- That problem can be solved with the use of second objective function:

  Minimum distance in the case of private transport from the center to each node.
- Minimum spanning tree problem includes variables y_{ij} :

  Represent how many times the road between *i*-th and *j*-th node is used.
- The objective represents the total traveled distance from center to individual nodes:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} y_{ij}$$

Problem formulation

$$f_1(\mathbf{x},\mathbf{y}) = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij} \rightarrow \min$$

$$f_2(\mathbf{x},\mathbf{y}) = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} y_{ij} \rightarrow \min$$

$$\sum_{j=2}^{n} x_{1j} = 0$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 2,3,\ldots n$$

$$\sum_{j=1}^{n} y_{ij} - \sum_{j=2}^{n} y_{ji} = 1, \qquad i = 2,3,\ldots n-1$$

$$0 \le y_{ij} \le (n-1)\, x_{ij}, \qquad i, j = 1,2,\ldots n$$

$$x_{ij} \in \{0,1\}, \qquad i, j = 1,2,\ldots n$$

Goal programming problem

- The problem can be solved as goal programming problem.

- To solve the problem it is needed to establish the weights for both criteria $\lambda_1 \geq 0, \lambda_2 \geq 0$

- One way of weighting is to determine the lowest and highest values for each criterion and on the base of relation it is possible to calculate the weights.

$$\frac{\lambda_2}{\lambda_1} = \frac{y_1^1 - y_1^0}{y_2^1 - y_2^0}$$

- Solving the problem by using the percentage deviations.

Formulation using L_1-metric

$$f(\mathbf{x},\mathbf{y},\mathbf{o}) = \lambda_1 o_1^+ + \lambda_2 o_2^+ \to \min$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_{ij} - y_1^0 o_1^+ \le y_1^0$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} y_{ij} - y_2^0 o_2^+ \le y_2^0$$

$$\sum_{j=2}^{n} x_{1j} = 0$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 2,3,\ldots n$$

$$\sum_{j=1}^{n} y_{ij} - \sum_{j=2}^{n} y_{ji} = 1, \qquad i = 2,3,\ldots n-1$$

$$0 \le y_{ij} \le (n-1) x_{ij}, \qquad i,j = 1,2,\ldots n$$

$$x_{ij} \in \{0,1\}, \qquad i,j = 1,2,\ldots n$$

$$o_1^+, o_2^+ \ge 0$$

Formulation using L_∞-metric

$$f(\mathbf{x},\mathbf{y},\alpha)=\alpha \rightarrow \min$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}x_{ij} - \frac{y_1^0}{\lambda_1}\alpha \le y_1^0$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}y_{ij} - \frac{y_2^0}{\lambda_2}\alpha \le y_2^0$$

$$\sum_{j=2}^{n} x_{1j} = 0$$

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 2,3,\ldots n$$

$$\sum_{j=1}^{n} y_{ij} - \sum_{j=2}^{n} y_{ji} = 1, \qquad i = 2,3,\ldots n-1$$

$$0 \le y_{ij} \le (n-1)\,x_{ij}, \qquad i,j = 1,2,\ldots n$$

$$x_{ij} \in \{0,1\}, \qquad i,j = 1,2,\ldots n$$

$$\alpha \ge 0$$