Using Goal Programming for Modeling Location Problems

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Location Problems

Two basic conception were developed for solving so that problems:

• first conception is aimed for covering the whole population with minimal number of service channels – *Location Set Covering Problem – LSCP,*

• the aim of second conception is to maximize the covering of population with limited number of service channels – *Maximal Covering Location Problem – MCLP.*

Population in Slovakia

- Slovak Republic (2001)
  - 5 378 511 inhabitants
  - 2 916 communes
  - 138 cities
  - 3 021 964 inhabitants (about 56%) was living in the cities

Goals of models

1. Model - shortest path when number of location centres is given
2. Model - minimal distance needed to travel when number of location centres is given
3. Model - combination problem of model 1 and model 2

Input data

- *n* – number of cities in Slovak Republic (138),
- ***D*** *(n x n)* – minimal distance matrix (elements represent the minimal distance from city *i* to city *j*)
- *p* – number of location centres
- b_j – number of inhabitants of *j*-th city

Modelling of incinerators in Slovakia
Variables

- y_{ij} – binary variable that represents the use of the route from *i*-th city to *j*-th location centre for $i, j = 1, 2, \ldots n$,
- z – represents maximal distance all the location centres from all the cities
- x_i – binary variable that represents if the location centre is open (value 1) or not (value 0) in *i*-th city for $i = 1, 2, \ldots n$.

Model 1

- *goal – finding the minimal distance between location centres and all the cities*

$$f(x,y,z) = z \rightarrow \min$$

$$\sum_{i=1}^{n} y_{ij} = 1, \quad j = 1, 2, \ldots n$$

$$y_{ij} - x_i \leq 0, \quad i, j = 1, 2, \ldots n$$

$$\sum_{i=1}^{n} x_i = p$$

$$\sum_{i=1}^{n} d_{ij} y_{ij} - z \leq 0, \quad j = 1, 2, \ldots n$$

$$x_i, y_{ij} \in \{0,1\}, \ i, j = 1, 2, \ldots n$$

$$z \geq 0$$

Model 2

- *goal – finding minimal total distance needed to travel when maximal number of location centres is given*

$$f\left(x,y\right)=\sum_{i=1}^{n}\sum_{j=1}^{n}b_j d_{ij} y_{ij} \rightarrow \min$$

$$\sum_{i=1}^{n} y_{ij}=1, \quad j=1,2,\ldots n$$

$$y_{ij}-x_i \le 0, \quad i,j=1,2,\ldots n$$

$$\sum_{i=1}^{n} x_i = p$$

$$x_i, y_{ij} \in \left\{0,1\right\}, \quad i,j=1,2,\ldots n$$

Model 3

- *goal – combination problem of Model 1 and Model 2*

$$f_1(\mathbf{x},\mathbf{y},z) = z \rightarrow \min$$

$$f_2(\mathbf{x},\mathbf{y}) = \sum_{i=1}^{n}\sum_{j=1}^{n} b_j d_{ij} y_{ij} \rightarrow \min$$

$$\sum_{i=1}^{n} y_{ij} = 1, \quad j = 1, 2, \ldots n$$

$$y_{ij} - x_i \leq 0, \quad i, j = 1, 2, \ldots n$$

$$\sum_{i=1}^{n} x_i = p$$

$$\sum_{i=1}^{n} d_{ij} y_{ij} - z \leq 0, \quad j = 1, 2, \ldots n$$

$$x_j, y_{ij} \in \{0,1\}, \; i, j = 1, 2, \ldots n$$

$$z \geq 0$$

Model 3 – L_1-metric

$$f\left(\mathbf{x},\mathbf{y},\mathbf{z},\mathbf{o}\right) = \lambda_1 o_1^+ + \lambda_2 o_2^+ \rightarrow \min$$

$$z - y_1^0 o_1^+ \leq y_1^0$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} b_j d_{ij} y_{ij} - y_2^0 o_2^+ \leq y_2^0$$

$$\sum_{i=1}^{n} y_{ij} = 1, \quad j = 1, 2, \ldots n$$

$$y_{ij} - x_i \leq 0, \quad i, j = 1, 2, \ldots n$$

$$\sum_{i=1}^{n} x_i = p$$

$$\sum_{i=1}^{n} d_{ij} y_{ij} - z \leq 0, \quad j = 1, 2, \ldots n$$

$$x_j, y_{ij} \in \{0,1\}, \; i, j = 1, 2, \ldots n$$

$$z, o_1^+, o_2^+ \geq 0$$

Model 3 – L_∞-metric

$$\min\ f(\mathbf{x},\mathbf{y},z,\alpha)=\alpha$$

$$z-\frac{y_1^0}{\lambda_1}\alpha\le y_1^0$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}b_j d_{ij} y_{ij}-\frac{y_2^0}{\lambda_2}\alpha\le y_2^0$$

$$\sum_{i=1}^{n}y_{ij}=1,\quad j=1,2,\ldots n$$

$$y_{ij}-x_i\le 0,\quad i,j=1,2,\ldots n$$

$$\sum_{i=1}^{n}x_i=p$$

$$\sum_{i=1}^{n}d_{ij}y_{ij}-z\le 0,\quad j=1,2,\ldots n$$

$$x_j,y_{ij}\in\{0,1\},\ i,j=1,2,\ldots n$$

$$z,\alpha\ge 0$$

Solving of models

- *models were solved by GAMS*
- model 1: with given parameter p = 10
- model 2: with given parameter p = 10
- model 3: with given parameters p = 10 *and weights of each criteria is 0,5*

Results

- Model 1: parameter *p* = 10

- Dolný Kubín, Hanušovce nad Topľou, Ilava, Malacky, Michalovce, Nové Zámky, Rožňava, Spišská Nová Ves, Veľký Krtíš, Zvolen with maximal distance 65,5 km (Dolný Kubín – Čadca)

Results

- Model 2: parameter *p* = 10
- Bánovce nad Bebravou (Stará Turá, 56 km), Bratislava (Dunajská Streda, 48,5 km), Košice (Tornaľa, 98,5 km), Michalovce (Medzilaborce, 70 km), Poprad (Hnúšťa, 72,5 km), Prešov (Svidník, 57,5 km), Šurany (Štúrovo, 61,5), Trnava (Gbely, 68,5 km), Zvolen (Rimavská Sobota, 87 km), Žilina (Trstená, 99,2 km)
- after each collection center is given also the distant city from that the waste is transported as well as corresponding cost (named into brackets).
- total distance 71 899 602,8 km

Results

- Model 3: parameters *p* = 10 *and weights of each criteria is 0,5*
- Bratislava, Michalovce, Poprad, Prešov, Rimavská Sobota, Ružomberok, Šurany, Vrbové, Zvolen, Žilina
- L_1 metric: Percentage deviation from goal 1 (minimal distance) is 6,9 %. Percentage deviation from goal 1 (total minimal distance) is 10,3 %. Value of objective function (weighted percentage deviation) is 8,6 %.
- L_∞ metric: Value of objective function (maximal deviation) is 5,2 %.

Modelling of location centres in Slovakia
Results – Figure (Model 3)